NEW TABLES

FOR

GREAT CIRCLE SAILING,

BY

A. H. DEICHMANN.

George Davidson

LONDON:

TRÜBNER & CO., 60, PATERNOSTER ROW.

1857.

The single sheets of the MANUSCRIPT here presented, advocating principles which are the result of the experience of practical seamen gained both on board of steamers and sailing vessels, have been sent to Mr. W. H. Prior to London, who has written to the Author the following letter.

„11 King Street, Tower Hill, London 24th March 1856.

„Dear Sir,

„I have with great attention examined your New Tables with the method of their application to Great Circle Sailing, and it affords me sincere pleasure to state that in my humble opinion, your work is decidedly superior to every other I have seen on the same subject.

„In addition to the advantages which in your Advertisement to the New Tables you have so clearly and truly shewn them to possess over those of Mr. Towson, I may add, that Towson's Tables in many instances produce very erroneous results, and Russel's Diagrams, though very beautifully got up and possessing much merit, are from the mode of their application not likely to be generally used by nautical men, while by your New Tables which are of easy application, the important Problem of sailing on a Great Circle is reduced to sailing on a series of Rhumbs which on the Chart are either Tangents or Chords, and their being carried to Quarter Points very greatly facilitates the laying down with accuracy the Course on the Chart.

„By means of your Blank Chart also the circle passing through two given points on the Earth's surface is readily determined by inspection only — to a degree of precision almost equal to that obtained by your very easy and practicable method of calculation, and I may here observe, that avoiding the application of a pair of compasses to your Blank Chart is an advantage which, though not perhaps frequently noticed, every one who has been in the habit of applying them to Towson's Linear Index will, I am certain, readily admit.

„The method of determining by construction the Great Circle passing through two given points — the Track — and the spherical Courses on the Great Circle, although it will perhaps be less generally used than the methods by Calculation and by Inspection, has all the merit of originality combined with correctness of result, qualities which equally apply to your Construction of the Limit of the Polar Tracks, an important ingredient in the General Problem of Great Circle Sailing.

„The Appendix which affords Mathematical proofs of the Construction of many of the most important Problems contained in the body of the work itself, cannot fail to prove truly acceptable to those whose abilities qualify them, to form a just estimate of its value, and I much regret that the limit of a Letter prevents my going into more minute details of that and of the other part of your work the merit of which, when known cannot fail to be duly appreciated by and to render it a valuable acquisition to the Practical Navigator.

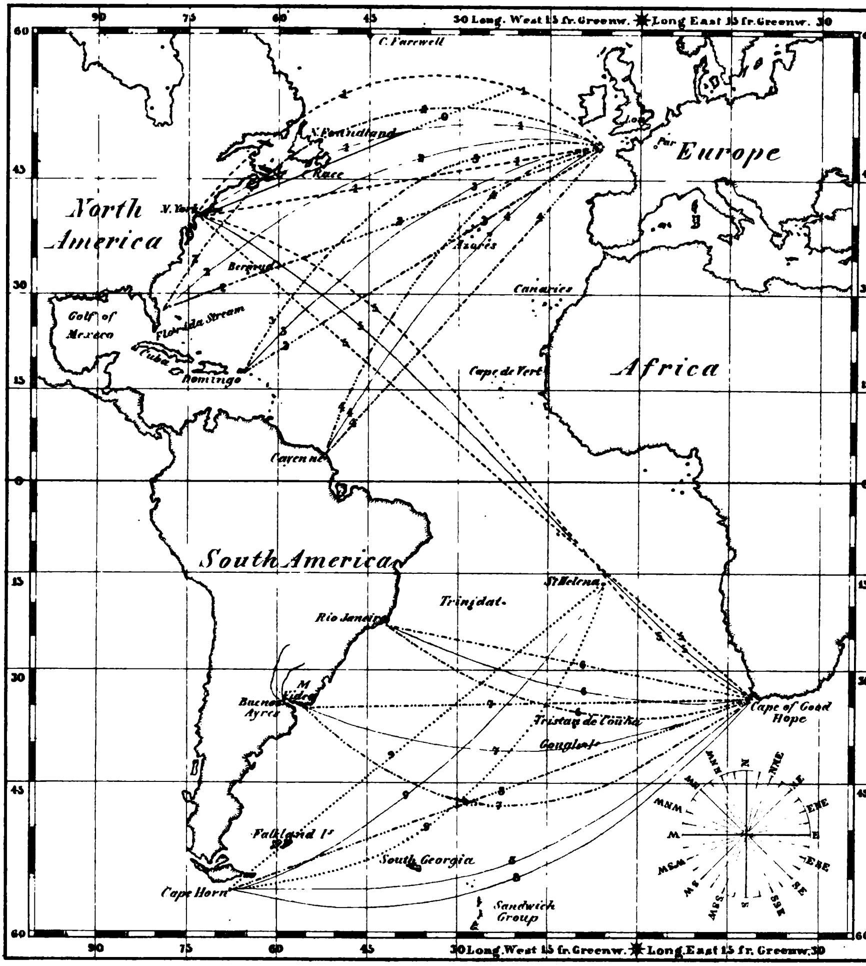
„I remain

„Yours very truly,

„W. H. Prior,
„Examiner of Navigation
to the Trinity House and Christ's Hospital.“

„Major Deichmann
Hanover.“

No. 1.



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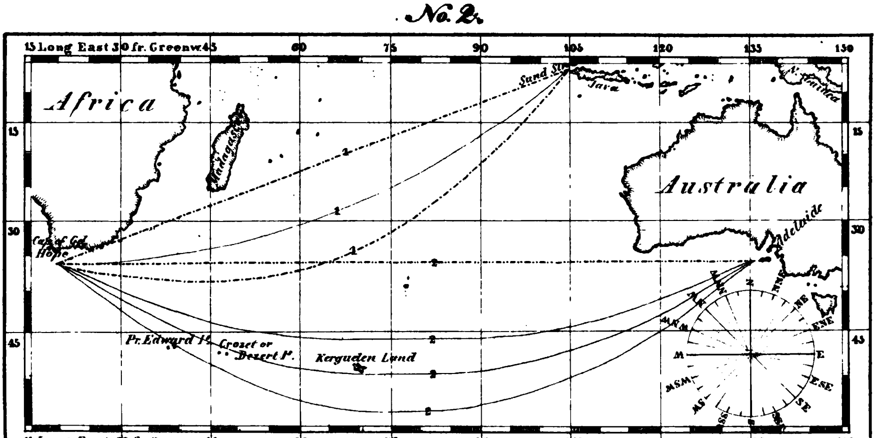

To the New Tables for Great Circle Sailing By Arnold H. Deichmann 1856.

George Davidson
1868

NEW TABLES

TO

FACILITATE THE PRACTICE

OF

GREAT CIRCLE SAILING,

TOGETHER WITH

AN APPLICATION

OF

THE THEORY OF THE GREAT CIRCLE ON THE GLOBE

TO

THE SAILING,

AND

AN APPENDIX,

CONTAINING SOME MATHEMATICAL DEMONSTRATIONS.

ACCOMPANIED BY

A SCALE OF GREAT CIRCLES ON A BLANK CHART,

TO DETERMINE WITHOUT CALCULATION
THE GREAT CIRCLE WHICH PASSES THROUGH TWO GIVEN PLACES
AND TO SHEW THE PLACES
AT WHICH THE SPHERICAL COURSES EXPRESSED IN FOURTHS OF THE POINT TAKE PLACE
ON THE GREAT CIRCLE'S ARC BETWEEN THE TWO GIVEN PLACES.

BY

A. H. DEICHMANN.

LONDON:
TRÜBNER & CO., 60, PATERNOSTER ROW.
1857.

In memoriam
George Davidson
1825 - 1911.

ADVERTISEMENT.

The theoretical features of the System of Great Circle Sailing have been known almost as long as navigation itself. The first treatise on Navigation, by Numez 1537, explains this system besides Mercator Sailing, and other classical Navigators after him advocated, already in the same century, the adoption of Great Circle Sailing instead of Mercator Sailing. But a more general practice of great circle sailing belongs to modern times, it is to be reckoned from the publication of *Mr. J. T. Towson's Tables to facilitate the practice of Great Circle Sailing.*

Those Tables, as published by the Admiralty, give the Latitudes, Spherical Courses and Distances on great circles of the globe, corresponding to each degree of Longitude reckoned from the Meridian of the Circle's Vertex. Although they determine stages each of which, having a definite direction, does not differ in length from the respective part of the great circle's arc between two given places, they are too short for practice and the Changing of the Course at their extremities is too small for being convenient.

The new Tables give the Latitudes, Longitudes from the Intersection of the Great Circle with the Equator, and Distances from it corresponding to each eighth of the Point of the Spherical Course. They determine stages as long as possible without giving a circuitous route, which is mathematically demonstrated; and besides, they admit of two ways for doing so, on Rhumbs being tangents on the Chart and on those which are chords on the Chart, and for both ways the Changing of the Course at the extremity of each stage is most convenient in practice, because these given Courses are expressed in fourths of the Point.

Each of Mr. Towson's Tables belongs to one great circle; each of the New Tables not only to that circle but to the next of the System and to all intermediate between those two, determining their Lats. and Longs. corresponding to the Courses adopted.

The elements to determine the Great Circle which passes through two given places require particular examination on „Mer. in“, „Equ. in“, „both out“, when employing Mr. Towson's Tables; whereas such an examination is not requisite, when employing the New Tables in which the origin of the Longs. is adopted in one of the common intersections of the System's Circles. The latter is of great importance to simplify the use of Great Circle Tables and has not been usual till now. —

Mr. Towson's Tables are accompanied by a „Linear Index“ containing a System of theoretical curves crossed by parallel straight lines to determine without calculation the elements to find the great circle which is the nearest in those Tables to that circle which passes through two given places. But this Linear Index itself does not give an idea of the great circle's arc between the given places. *Mr. R. Russel's Diagram* gives such an idea still clearer and more convenient for practice than a terrestrial globe. But both do not give exactly enough the elements to find the great circle required in the Tables, if they are constructed by the usual measure. To construct that Diagram by a larger measure, than is employed, would be useless on account of its extension; and that Diagram does not shew the spherical Courses directly.

The Great Circle Scale accompanying the New Tables answers all of those requisitions. It is constructed large enough to determine the elements required, which is made possible, because the use has been reduced to the representation of a fourth of the surface of the globe, wheras Mr. Russel's Diagram requires for its use the representation of the whole surface of the globe. The idea of the great circle's arc required is clearly represented by that Scale, and the spherical Courses to be expressed in fourths of the Point are indicated on the plate. —

Of the Rules given to find the proper longitudes being elements to determine the great circle required in the Tables, that by calculation is shorter than the usual (Napier's Analogy), and that by construction is new. Both are elementarily demonstrated in the Appendix. —

Great Circle Sailing includes now not only the case in which the ship is moving along the respective great circle's arc, but also along routes on the sides of this arc determined with respect to it between the same two given places.

Lieut. Raper mentions those comparisons shortly in the part „*Navigating the Ship*“ of his generally known work, „*The Practice of Navigation*“ under „*Shaping the Course on a Great Circle*“, without giving special rules for the determination of those tracks. *J. Masters Share's Great Circle Tables for the North Atlantic* constructed to determine by inspection the Great Circle Course and the Course by the Rhumb both between the two given places contain a rule to find by means of those two Courses by a simple calculation a Course on the polar side of the great circle's arc corresponding to the Rhumb Course and called by him „Polar Course“, but the track itself is not determined by that rule.

To determine those tracks and to base them upon the respective great circle's arc is one of the objects of the investigation in the following pages. The given construction of the Limit of the Polar Tracks is new and treated both by common nautical rules and spherical trigonometry.

Hanover. March, 1856. A. H. D.

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(1) $(\sin a'' + \sin a') : (\sin a'' - \sin a') :: \tan \frac{a''+a'}{2} : \tan \frac{a''-a'}{2}$ or, $\frac{\sin a'' + \sin a'}{\sin a'' - \sin a'} = \frac{\tan \frac{a''+a'}{2}}{\tan \frac{a''-a'}{2}}$

(2) $(\tan b'' + \tan b') : (\tan b'' - \tan b') :: \sin(b''+b') : \sin(b''-b')$ or, $\frac{\tan b'' + \tan b'}{\tan b'' - \tan b'} = \frac{\sin(b''+b')}{\sin(b''-b')}$. 82—83

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ERRATA.

Besides the erratum bound into the book, page 45, the following errata might lead to errors,

page	*line*	*for*	*read*
15	2 from bot.	if the great circle	of the great circle
30	1	TABLES No. 51	TABLE No. 51
44	1	$(B)(A')$	$(B)(A'')$
44	5 from bot.	, the base of the Model along	, the base of the Model, along
45	10	the longitudes from	the longitude from
48	(in Fig. 21, on *CD*)	N''	N'
48	15 from bot.	upon *C*	upon *EC*
54	7 ———	Art. 62	Art. 62—64
59	5	Art. 97	Art. 67.
66	27	cutting points	cutting point
66	8 from bot.	with *1* that	with *1*, that
67	18	D. Long.; the Rhumb	D. Long., the Rhumb
68	8 from bot.	intervals; into	intervals into
75	9 ———	Cape Breten	Cape Breton
76	4 ———	so such	so much
80	16	Lat. req. of Pa.	Long. req. of Pa.

TABLES

FOR

GREAT CIRCLE SAILING.

The Inclin. to the Equat. is = 1° — **The Latitude of Vertex is = 1°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
8	1 0	1 0	90		0 0	
*	0 0	0 0	0 0	& 180 0	0 0	5400

* The Course in the Inters. with the Equat. is equal to 89°.

The Inclin. to the Equat. is = 2° — **The Latitude of Vertex is = 2°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	2 0	1 0	90		0 0	
$7\frac{7}{8}$	1 25	1 14	45 19	134 41	16 43	2680
*	0 0	0 0	0 0	180 0	0 0	2720

* The Course in the Inters. with the Equat. is equal to 88°.

The Inclin. to the Equat. is = 3° — **The Latitude of Vertex is = 3°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	3 0	1 0	90		0 0	
$7\frac{7}{8}$	2 39	1 6	62 2	117 58	7 22	1676
$7\frac{3}{4}$	1 3	1 48	20 21	159 39	24 57	2501
*	0 0	0 0	0 0	180 0	0 0	1223

* The Course in the Inters. with the Equat. is equal to 87°.

The Inclin. to the Equat. is = 4° — **The Latitude of Vertex is = 4°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	4 0	1 0	90		0 0	
$7\frac{7}{8}$	3 45	1 3	69 24	110 36	4 15	1233
$7\frac{3}{4}$	2 51	1 17	45 18	134 42	10 26	1445
*	0 0	0 0	0 0	180 0	0 0	2722

* The Course in the Inters. with the Equat. is equal to 86°.

The Inclin. to the Equat. is = 5° — **The Latitude of Vertex is = 5°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	5 0	1 0	90		0 0	
$7\frac{7}{8}$	4 48	1 2	73 39	106 21	2 46	978
$7\frac{3}{4}$	4 8	1 10	55 44	124 16	6 16	1072
$7\frac{5}{8}$	2 42	1 34	32 26	147 34	12 50	1398
*	0 0	0 0	0 0	180 0	0 0	1952

* The Course in the Inters. with the Equat. is equal to 85°.

The Inclin. to the Equat. is = 6° — **The Latitude of Vertex is = 6°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	6 0	1 0	90		0 0	
$7\frac{7}{8}$	5 50	1 1	76 25	103 35	1 58	811
$7\frac{3}{4}$	5 18	1 7	62 0	118 0	4 15	861
$7\frac{5}{8}$	4 16	1 19	45 16	134 44	7 36	1002
$7\frac{1}{2}$	2 5	2 5	20 20	159 40	16 8	1500
*	0 0	0 0	0 0	180 0	0 0	1226

* The Course in the Inters. with the Equat. is equal to 84°.

The Inclin. to the Equat. is = 7° — **The Latitude of Vertex is = 7°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	7 0	1 0	90		0 0	
$7\frac{7}{8}$	6 51	1 2	78 23	101 37	1 28	692
$7\frac{3}{4}$	6 25	1 5	66 15	113 45	3 6	723
$7\frac{5}{8}$	5 35	1 13	52 52	127 8	5 13	800
$7\frac{1}{2}$	4 10	1 32	36 28	143 32	8 46	985
*	0 0	0 0	0 0	180 0	0 0	2200

* The Course in the Inters. with the Equat. is equal to 83°.

The Inclin. to the Equat. is = 8° — **The Latitude of Vertex is = 8°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	8 0	1 0	90		0 0	
$7\frac{7}{8}$	7 53	1 0	79 51	100 9	1 7	604
$7\frac{3}{4}$	7 30	1 3	69 21	110 39	2 22	624
$7\frac{5}{8}$	6 48	1 9	58 5	121 55	3 52	672
$7\frac{1}{2}$	5 42	1 20	45 14	134 46	5 58	769
$7\frac{3}{8}$	3 50	1 48	28 25	151 35	10 6	1012
*	0 0	0 0	0 0	180 0	0 0	1719

* The Course in the Inters. with the Equat. is equal to 82°.

The Inclin. to the Equat. is = 9° — **The Latitude of Vertex is = 9°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	9 0	1 0	90		0 0	
$7\frac{7}{8}$	8 53	1 1	80 58	99 2	0 55	535
$7\frac{3}{4}$	8 33	1 3	71 43	108 17	1 52	549
$7\frac{5}{8}$	7 57	1 7	61 57	118 3	2 59	581
$7\frac{1}{2}$	7 2	1 15	51 12	128 48	4 26	642
$7\frac{3}{8}$	5 38	1 30	38 31	141 29	6 40	762
$7\frac{1}{4}$	3 9	2 14	20 17	159 43	12 3	1100
*	0 0	0 0	0 0	180 0	0 0	1231

* The Course in the Inters. with the Equat. is equal to 81°.

10 — The Inclin. to the Equat. is $= 10°$ — The Latitude of Vertex is $= 10°$

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	10 0	1 0	90		0 0	
7 7/8	9 54	1 1	81 53	98 7	0 44	480
7 3/4	9 36	1 2	73 35	106 25	1 31	491
7 5/8	9 4	1 6	64 56	115 4	2 23	513
7 1/2	8 17	1 11	55 38	124 22	3 27	553
7 3/8	7 8	1 21	45 11	134 49	4 55	626
7 1/4	5 23	1 42	32 20	147 40	7 24	773
7 1/8	1 46	3 10	10 5	169 55	16 17	1349
*	0 0	0 0	0 0	180 0	0 0	615

* The Course in the Inters. with the Equat. is equal to 80°.

11 — The Inclin. to the Equat. is $= 11°$ — The Latitude of Vertex is $= 11°$

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	11 0	1 0	90		0 0	
7 7/8	10 55	1 0	82 37	97 23	0 36	435
7 3/4	10 38	1 2	75 6	104 54	1 15	443
7 5/8	10 10	1 5	67 19	112 41	1 58	460
7 1/2	9 28	1 9	59 5	120 55	2 47	489
7 3/8	8 29	1 16	50 6	129 54	3 50	536
7 1/4	7 5	1 29	39 44	140 16	5 23	621
7 1/8	4 56	1 58	26 22	153 38	8 19	809
*	0 0	0 0	0 0	180 0	0 0	1607

* The Course in the Inters. with the Equat. is equal to 79°.

12 — The Inclin. to the Equat. is $= 12°$ — The Latitude of Vertex is $= 12°$

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	12 0	1 0	90		0 0	
7 7/8	11 55	1 1	83 13	96 47	0 31	398
7 3/4	11 40	1 2	76 21	103 39	1 3	404
7 5/8	11 15	1 4	69 17	110 43	1 38	416
7 1/2	10 37	1 7	61 52	118 8	2 18	438
7 3/8	9 45	1 13	53 56	126 4	3 6	472
7 1/4	8 34	1 22	45 7	134 53	4 10	527
7 1/8	6 54	1 38	34 41	145 19	5 51	628
7	4 12	2 21	20 14	159 46	9 38	879
*	0 0	0 0	0 0	180 0	0 0	1238

* The Course in the Inters. with the Equat. is equal to 78°.

13 — The Inclin. to the Equat. is $= 13°$ — The Latitude of Vertex is $= 13°$

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	13 0	1 0	90		0 0	
7 7/8	12 56	1 0	83 44	96 16	0 27	366
7 3/4	12 42	1 1	77 24	102 36	0 54	371
7 5/8	12 19	1 3	70 55	109 5	1 23	381
7 1/2	11 44	1 6	64 10	115 50	1 56	397
7 3/8	10 58	1 10	57 2	122 58	2 34	423
7 1/4	9 56	1 17	49 17	130 43	3 23	461
7 1/8	8 32	1 28	40 32	139 28	4 30	525
7	6 33	1 50	29 52	150 8	6 23	646
6 7/8	3 0	3 2	13 6	166 54	11 59	1024
*	0 0	0 0	0 0	180 0	0 0	806

* The Course in the Inters. with the Equat. is equal to 77°.

14 — The Inclin. to the Equat. is $= 14°$ — The Latitude of Vertex is $= 14°$

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	14 0	1 0	90		0 0	
7 7/8	13 56	1 0	84 11	95 49	0 23	339
7 3/4	13 43	1 1	78 18	101 42	0 46	343
7 5/8	13 22	1 2	72 18	107 42	1 11	351
7 1/2	12 50	1 6	66 6	113 54	1 39	363
7 3/8	12 8	1 9	59 36	120 24	2 10	383
7 1/4	11 13	1 14	52 40	127 20	2 48	412
7 1/8	10 0	1 22	45 2	134 58	3 38	455
7	8 23	1 36	36 15	143 45	4 50	529
6 7/8	6 2	2 5	25 5	154 55	7 5	680
*	0 0	0 0	0 0	180 0	0 0	1545

* The Course in the Inters. with the Equat. is equal to 76°.

15 — The Inclin. to the Equat. is $= 15°$ — The Latitude of Vertex is $= 15°$

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	15 0	1 0	90		0 0	
7 7/8	14 56	1 0	84 34	95 26	0 20	315
7 3/4	14 44	1 1	79 4	100 56	0 41	319
7 5/8	14 24	1 3	73 29	106 31	1 2	325
7 1/2	13 56	1 4	67 45	112 15	1 25	335
7 3/8	13 17	1 8	61 46	118 14	1 52	350
7 1/4	12 27	1 12	55 28	124 32	2 22	373
7 1/8	11 22	1 19	48 40	131 20	3 0	405
7	9 59	1 28	41 5	138 55	3 52	454
6 7/8	8 7	1 45	32 10	147 50	5 11	540
6 3/4	5 16	2 27	20 9	159 51	8 1	736
*	0 0	0 0	0 0	180 0	0 0	1248

* The Course in the Inters. with the Equat. is equal to 75°.

The Inclin. to the Equat. is = 16° — 16 — The Latitude of Vertex is = 16°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	16 0	1 0	90		0 0	295
7 7/8	15 56	1 1	84 54	95 6	0 17	297
7 3/4	15 45	1 1	79 45	100 15	0 35	302
7 5/8	15 27	1 2	74 31	105 29	0 55	311
7 1/2	15 0	1 4	69 10	110 50	1 15	324
7 3/8	14 25	1 6	63 38	116 22	1 37	340
7 1/4	13 39	1 10	57 50	122 10	2 3	365
7 1/8	12 41	1 15	51 40	128 20	2 33	401
7	11 27	1 23	44 57	135 3	3 12	458
6 7/8	9 52	1 35	37 21	142 39	4 7	559
6 3/4	7 43	1 56	28 10	151 50	7 37	837
6 5/8	4 8	3 0	14 37	165 23	9 34	911
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 74°.

The Inclin. to the Equat. is = 17° — 17 — The Latitude of Vertex is = 17°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	17 0	1 0	90		0 0	276
7 7/8	16 57	1 0	85 11	94 49	0 16	279
7 3/4	16 46	1 1	80 20	99 40	0 32	283
7 5/8	16 29	1 2	75 26	104 34	0 48	290
7 1/2	16 4	1 4	70 25	109 35	1 5	300
7 3/8	15 31	1 6	65 15	114 45	1 25	313
7 1/4	14 49	1 8	59 53	120 7	1 46	334
7 1/8	13 56	1 13	54 13	125 47	2 12	360
7	12 50	1 19	48 9	131 51	2 42	401
6 7/8	11 27	1 27	41 28	138 32	3 23	465
6 3/4	9 39	1 42	33 47	146 13	4 23	590
6 5/8	7 8	2 11	24 11	155 49	6 9	1079
6 1/2	2 5	4 16	6 51	173 9	13 12	430
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 73°.

The Inclin. to the Equat. is = 18° — 18 — The Latitude of Vertex is = 18°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	18 0	1 0	90		0 0	260
7 7/8	17 57	1 0	85 27	94 33	0 14	262
7 3/4	17 47	1 1	80 52	99 8	0 28	265
7 5/8	17 31	1 2	76 14	103 46	0 42	272
7 1/2	17 8	1 3	71 30	108 30	0 59	280
7 3/8	16 37	1 5	66 40	113 20	1 15	291
7 1/4	15 57	1 8	61 39	118 21	1 34	307
7 1/8	15 9	1 11	56 25	123 35	1 54	328
7	14 9	1 16	50 51	129 9	2 20	358

Continuation of the Great Circle No. 18.

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
6 7/8	12 54	1 23	44 51	135 9	2 51	403
6 3/4	11 21	1 33	38 10	141 50	3 34	478
6 5/8	9 19	1 51	30 20	149 40	4 40	636
6 1/2	6 21	2 31	20 3	159 57	6 52	1260
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 72°.

The Inclin. to the Equat. is = 19° — 19 — The Latitude of Vertex is = 19°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	19 0	1 0	90		0 0	245
7 7/8	18 57	1 0	85 41	94 19	0 12	247
7 3/4	18 48	1 1	81 20	98 40	0 25	250
7 5/8	18 33	1 1	76 56	103 4	0 39	255
7 1/2	18 11	1 2	72 29	107 31	0 52	262
7 3/8	17 42	1 4	67 55	112 5	1 7	272
7 1/4	17 5	1 7	63 13	116 47	1 23	285
7 1/8	16 20	1 10	58 19	121 41	1 42	301
7	15 25	1 14	53 11	126 49	2 2	325
6 7/8	14 17	1 20	47 42	132 18	2 28	358
6 3/4	12 54	1 28	41 44	138 16	3 0	409
6 5/8	11 10	1 40	35 0	145 0	3 45	497
6 1/2	8 52	2 2	26 55	153 5	5 1	711
6 3/8	5 16	2 59	15 32	164 28	7 57	983
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 71°.

The Inclin. to the Equat. is = 20° — 20 — The Latitude of Vertex is = 20°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	20 0	1 0	90		0 0	232
7 7/8	19 57	1 0	85 53	94 7	0 11	233
7 3/4	19 49	1 0	81 45	98 15	0 23	237
7 5/8	19 34	1 1	77 35	102 25	0 34	240
7 1/2	19 13	1 3	73 21	106 39	0 47	246
7 3/8	18 46	1 4	69 2	110 58	1 0	255
7 1/4	18 12	1 6	64 36	115 24	1 14	265
7 1/8	17 30	1 9	60 1	119 59	1 29	280
7	16 39	1 12	55 13	124 47	1 48	298
6 7/8	15 37	1 17	50 10	129 50	2 9	323
6 3/4	14 22	1 23	44 44	135 16	2 35	361
6 5/8	12 50	1 33	38 45	141 15	3 9	417
6 1/2	10 54	1 47	31 56	148 4	3 58	524
6 3/8	8 15	2 16	23 29	156 31	5 26	835
6 1/4	3 36	3 51	9 56	170 4	10 0	634
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 70°.

The Inclin. to the Equat. is = 21° — The Latitude of Vertex is = 21°

Course	Lat.		diff.		Long. from the Intrs.				diff.		Dist.
Points	°	′	°	′	°	′ &	°	′	°	′	Miles
8	21	0	1	0	90				0	0	220
$7^7/_8$	20	57	1	0	86	4	93	56	0	11	221
$7^3/_4$	20	49	1	1	82	8	97	52	0	20	224
$7^5/_8$	20	35	1	2	78	9	101	51	0	31	227
$7^1/_2$	20	16	1	2	74	8	105	52	0	42	233
$7^3/_8$	19	50	1	4	70	2	109	58	0	54	239
$7^1/_4$	19	18	1	6	65	50	114	10	1	6	248
$7^1/_8$	18	39	1	7	61	30	118	30	1	21	260
7	17	51	1	11	57	1	122	59	1	36	276
$6^7/_8$	16	54	1	15	52	19	127	41	1	53	296
$6^3/_4$	15	45	1	21	47	19	132	41	2	15	324
$6^5/_8$	14	23	1	27	41	54	138	6	2	42	365
$6^1/_2$	12	41	1	39	35	54	144	6	3	18	430
$6^3/_8$	10	31	1	56	28	55	151	5	4	13	563
$6^1/_4$	7	27	2	34	19	56	160	4	6	0	1274
*	0	0	0	0	0	0	180	0	0	0	

* The Course in the Inters. with the Equat. is equal to 69°.

The Inclin. to the Equat. is = 22° — The Latitude of Vertex is = 22°

Course	Lat.		diff.		Long. from the Intrs.				diff.		Dist.
Points	°	′	°	′	°	′ &	°	′	°	′	Miles
8	22	0	1	0	90				0	0	209
$7^7/_8$	21	57	1	1	86	15	93	45	0	9	210
$7^3/_4$	21	50	1	0	82	28	97	32	0	19	212
$7^5/_8$	21	37	1	1	78	40	101	20	0	29	216
$7^1/_2$	21	18	1	2	74	50	105	10	0	38	219
$7^3/_8$	20	54	1	3	70	56	109	4	0	49	226
$7^1/_4$	20	24	1	5	66	56	113	4	1	1	234
$7^1/_8$	19	46	1	7	62	51	117	9	1	12	244
7	19	2	1	10	58	37	121	23	1	26	256
$6^7/_8$	18	9	1	13	54	12	125	48	1	42	273
$6^3/_4$	17	6	1	17	49	34	130	26	1	59	295
$6^5/_8$	15	50	1	24	44	36	135	24	2	21	326
$6^1/_2$	14	20	1	32	39	12	140	48	2	49	371
$6^3/_8$	12	27	1	45	33	8	146	52	3	28	449
$6^1/_4$	10	1	2	7	25	56	154	4	4	30	622
$6^1/_8$	6	24	2	59	16	7	163	53	6	48	1038
*	0	0	0	0	0	0	180	0	0	0	

* The Course in the Inters. with the Equat. is equal to 68°.

The Inclin. to the Equat. is = 23° — The Latitude of Vertex is = 23°

Course	Lat.		diff.		Long. from the Intrs.				diff.		Dist.
Points	°	′	°	′	°	′ &	°	′	°	′	Miles
8	23	0	1	0	90				0	0	199
$7^7/_8$	22	58	1	0	86	24	93	36	0	8	200
$7^3/_4$	22	50	1	1	82	47	97	13	0	17	201
$7^5/_8$	22	38	1	1	79	9	100	51	0	26	205
$7^1/_2$	22	20	1	2	75	28	104	32	0	35	209
$7^3/_8$	21	57	1	3	71	45	108	15	0	44	213
$7^1/_4$	21	29	1	4	67	57	112	3	0	54	221
$7^1/_8$	20	53	1	7	64	3	115	57	1	6	229
7	20	12	1	8	60	3	119	57	1	17	240
$6^7/_8$	19	22	1	12	55	54	124	6	1	30	253
$6^3/_4$	18	23	1	16	51	33	128	27	1	46	271
$6^5/_8$	17	14	1	21	46	57	133	3	2	4	296
$6^1/_2$	15	52	1	27	42	1	137	59	2	27	329
$6^3/_8$	14	12	1	37	36	36	143	24	2	56	381
$6^1/_4$	12	8	1	52	30	26	149	34	2	39	473
$6^1/_8$	9	23	2	20	22	55	157	5	2	51	723
6	4	54	3	41	11	39	168	21	8	9	757
*	0	0	0	0	0	0	180	0	0	0	

* The Course in the Inters. with the Equat. is equal to 67°.

The Inclin. to the Equat. is = 24° — The Latitude of Vertex is = 24°

Course	Lat.		diff.		Long. from the Intrs.				diff.		Dist.
Points	°	′	°	′	°	′ &	°	′	°	′	Miles
8	24	0	1	0	90				0	0	190
$7^7/_8$	23	58	1	0	86	32	93	28	0	8	190
$7^3/_4$	23	51	1	0	83	4	96	56	0	16	192
$7^5/_8$	23	39	1	1	79	35	100	25	0	24	195
$7^1/_2$	23	22	1	2	76	3	103	57	0	32	198
$7^3/_8$	23	0	1	3	72	29	107	31	0	41	203
$7^1/_4$	22	33	1	4	68	51	111	9	0	50	208
$7^1/_8$	22	0	1	6	65	9	114	51	0	59	216
7	21	20	1	8	61	20	118	40	1	10	225
$6^7/_8$	20	34	1	10	57	24	122	36	1	22	237
$6^3/_4$	19	39	1	14	53	19	126	41	1	35	252
$6^5/_8$	18	35	1	18	49	1	130	59	1	51	271
$6^1/_2$	17	19	1	24	44	28	135	32	2	9	297
$6^3/_8$	15	49	1	32	39	32	140	28	2	33	335
$6^1/_4$	14	0	1	43	34	5	145	55	3	3	395
$6^1/_8$	11	43	2	1	27	46	152	14	3	51	505
6	8	35	2	37	19	48	160	12	5	19	968
$5^7/_8$	2	11	5	21	4	55	175	5	11	34	323
*	0	0	0	0	0	0	180	0	0	0	

* The Course in the Inters. with the Equat. is equal to 66°.

The Inclin. to the Equat. is = 25° — The Latitude of Vertex is = 25°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	25 0	1 0	90		0 0	181
$7^7/_8$	24 58	1 0	86 40	93 20	0 7	182
$7^3/_4$	24 51	1 0	83 20	96 40	0 14	183
$7^5/_8$	24 40	1 1	79 59	100 1	0 21	186
$7^1/_2$	24 24	1 2	76 35	103 25	0 30	188
$7^3/_8$	24 3	1 3	73 10	106 50	0 37	193
$7^1/_4$	23 37	1 4	69 41	110 19	0 46	198
$7^1/_8$	23 6	1 5	66 8	113 52	0 55	204
7	22 28	1 8	62 30	117 30	1 4	212
$6^7/_8$	21 44	1 10	58 46	121 14	1 15	222
$6^3/_4$	20 53	1 13	54 54	125 6	1 26	235
$6^5/_8$	19 53	1 17	50 52	129 8	1 39	251
$6^1/_2$	18 43	1 22	46 37	133 23	1 55	272
$6^3/_8$	17 21	1 28	42 5	137 55	2 14	300
$6^1/_4$	15 43	1 37	37 8	142 52	2 39	343
$6^1/_8$	13 44	1 50	31 37	148 23	3 12	409
6	11 12	2 11	25 7	154 53	4 5	556
$5^7/_8$	7 32	3 0	16 29	163 31	5 56	1085
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 65°.

The Inclin. to the Equat. is = 26° — The Latitude of Vertex is = 26°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	26 0	1 0	90		0 0	173
$7^7/_8$	25 58	1 0	86 47	93 13	0 7	174
$7^3/_4$	25 51	1 1	83 34	96 26	0 14	175
$7^5/_8$	25 41	1 1	80 20	99 40	0 20	177
$7^1/_2$	25 26	1 1	77 5	102 55	0 27	180
$7^3/_8$	25 6	1 2	73 47	106 13	0 34	183
$7^1/_4$	24 41	1 4	70 27	109 33	0 42	188
$7^1/_8$	24 11	1 5	67 3	112 57	0 50	194
7	23 36	1 6	63 34	116 26	0 59	201
$6^7/_8$	22 54	1 9	60 1	119 59	1 8	209
$6^3/_4$	22 6	1 11	56 20	123 40	1 19	220
$6^5/_8$	21 10	1 15	52 31	127 29	1 30	234
$6^1/_2$	20 5	1 19	48 32	131 28	1 43	250
$6^3/_8$	18 49	1 24	44 19	135 41	1 59	273
$6^1/_4$	17 20	1 31	39 47	140 13	2 19	306
$6^1/_8$	15 34	1 41	34 49	145 11	2 44	351
6	13 23	1 57	29 12	150 48	3 21	433
$5^7/_8$	10 32	2 24	22 25	157 35	4 23	631
$5^3/_4$	6 9	3 34	12 45	167 15	6 54	848
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 64°.

The Inclin. to the Equat. is = 27° — The Latitude of Vertex is = 27°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	27 0	1 0	90		0 0	166
$7^7/_8$	26 58	1 0	86 54	93 6	0 6	166
$7^3/_4$	26 52	1 0	83 48	96 12	0 12	167
$7^5/_8$	26 42	1 0	80 40	99 20	0 19	170
$7^1/_2$	26 27	1 1	77 32	102 28	0 25	172
$7^3/_8$	26 8	1 2	74 21	105 39	0 32	175
$7^1/_4$	25 45	1 3	71 9	108 51	0 38	179
$7^1/_8$	25 16	1 5	67 53	112 7	0 46	184
7	24 42	1 7	64 33	115 27	0 54	190
$6^7/_8$	24 3	1 8	61 9	118 51	1 2	198
$6^3/_4$	23 17	1 11	57 39	122 21	1 11	207
$6^5/_8$	22 25	1 13	54 1	125 59	1 22	218
$6^1/_2$	21 24	1 17	50 15	129 45	1 33	233
$6^3/_8$	20 13	1 22	46 18	133 42	1 46	251
$6^1/_4$	18 51	1 28	42 6	137 54	2 3	277
$6^1/_8$	17 15	1 36	37 33	142 27	2 24	310
6	15 20	1 47	32 33	147 27	2 51	364
$5^7/_8$	12 56	2 6	26 48	153 12	3 31	463
$5^3/_4$	9 43	2 40	19 39	160 21	4 45	775
$5^5/_8$	4 2	4 40	7 58	172 2	8 46	535
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 63°.

The Inclin. to the Equat. is = 28° — The Latitude of Vertex is = 28°

Course	Lat.	diff.	Long. f. om the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	28 0	1 0	90		0 0	159
$7^7/_8$	27 58	1 0	87 0	93 0	0 6	159
$7^3/_4$	27 52	1 1	84 0	96 0	0 11	160
$7^5/_8$	27 42	1 1	80 59	99 1	0 17	162
$7^1/_2$	27 28	1 2	77 57	102 3	0 23	165
$7^3/_8$	27 10	1 2	74 53	105 7	0 30	167
$7^1/_4$	26 48	1 3	71 47	108 13	0 36	171
$7^1/_8$	26 21	1 4	68 39	111 21	0 42	175
7	25 49	1 5	65 27	114 33	0 49	181
$6^7/_8$	25 11	1 8	62 11	117 49	0 57	187
$6^3/_4$	24 28	1 10	58 50	121 10	1 5	196
$6^5/_8$	23 38	1 12	55 23	124 37	1 14	205
$6^1/_2$	22 41	1 15	51 48	128 12	1 25	218
$6^3/_8$	21 35	1 20	48 4	131 56	1 37	233
$6^1/_4$	20 19	1 25	44 9	135 51	1 50	253
$6^1/_8$	18 51	1 31	39 57	140 3	2 7	279
6	17 7	1 41	35 24	144 36	2 29	319
$5^7/_8$	15 2	1 53	30 19	149 41	2 59	380
$5^3/_4$	12 23	2 16	24 24	155 36	3 44	503
$5^5/_8$	8 42	3 1	16 44	163 16	5 14	1128
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 62°.

The Inclin. to the Equat. is = 29° **29** The Latitude of Vertex is = 29°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	29 0	1 0	90		0 0	152
7 7/8	28 58	1 0	87 6	92 54	0 5	153
7 3/4	28 53	1 0	84 11	95 49	0 11	154
7 5/8	28 43	1 1	81 16	98 44	0 16	155
7 1/2	28 30	1 1	78 20	101 40	0 22	157
7 3/8	28 12	1 2	75 23	104 37	0 27	160
7 1/4	27 51	1 3	72 23	107 37	0 33	164
7 1/8	27 25	1 4	69 21	110 39	0 39	167
7	26 54	1 6	66 16	113 44	0 46	172
6 7/8	26 19	1 7	63 8	116 52	0 53	178
6 3/4	25 38	1 9	59 55	120 5	1 0	185
6 5/8	24 50	1 12	56 37	123 23	1 9	194
6 1/2	23 56	1 15	53 13	126 47	1 18	204
6 3/8	22 55	1 18	49 41	130 19	1 28	217
6 1/4	21 44	1 22	45 59	134 1	1 39	235
6 1/8	20 22	1 28	42 4	137 56	1 54	254
6	18 48	1 35	37 53	142 7	2 11	285
5 7/8	16 55	1 46	33 18	146 42	2 33	328
5 3/4	14 39	2 1	28 8	151 52	3 6	400
5 5/8	11 43	2 28	21 58	158 2	3 59	564
5 1/2	7 23	3 31	13 31	166 29	5 57	922
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 61°.

The Inclin. to the Equat. is = 30° **30** The Latitude of Vertex is = 30°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	30 0	1 0	90		0 0	146
7 7/8	29 58	1 0	87 11	92 49	0 5	147
7 3/4	29 53	1 0	84 22	95 38	0 10	147
7 5/8	29 44	1 0	81 32	98 28	0 15	149
7 1/2	29 31	1 1	78 42	101 18	0 20	151
7 3/8	29 14	1 2	75 50	104 10	0 25	153
7 1/4	28 54	1 2	72 56	107 4	0 31	156
7 1/8	28 29	1 4	70 0	110 0	0 37	160
7	28 0	1 5	67 2	112 58	0 42	164
6 7/8	27 26	1 6	64 1	115 59	0 48	170
6 3/4	26 47	1 8	60 55	119 5	0 56	175
6 5/8	26 2	1 10	57 46	122 14	1 3	184
6 1/2	25 11	1 13	54 31	125 29	1 11	192

Continuation of the Great Circle №. 30.

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
6 3/8	24 13	1 16	51 9	128 51	1 20	204
6 1/4	23 6	1 20	47 38	132 22	1 31	218
6 1/8	21 50	1 25	43 58	136 2	1 42	235
6	20 23	1 31	40 4	139 56	1 57	258
5 7/8	18 41	1 40	35 51	144 9	2 16	291
5 3/4	16 40	1 51	31 14	148 46	2 39	340
5 5/8	14 11	2 9	25 57	154 3	3 15	427
5 1/2	10 54	2 42	19 28	160 32	4 17	665
5 3/8	5 32	4 21	9 40	170 20	7 12	668
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 60°.

The Inclin. to the Equat. is = 31° **31** The Latitude of Vertex is = 31°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	31 0	1 0	90		0 0	140
7 7/8	30 58	1 0	87 16	92 44	0 5	141
7 3/4	30 53	1 0	84 32	95 28	0 9	142
7 5/8	30 44	1 1	81 47	98 13	0 14	143
7 1/2	30 32	1 1	79 2	100 58	0 18	145
7 3/8	30 16	1 2	76 15	103 45	0 24	147
7 1/4	29 56	1 3	73 27	106 33	0 29	149
7 1/8	29 33	1 3	70 37	109 23	0 34	153
7	29 5	1 4	67 44	112 16	0 40	157
6 7/8	28 32	1 6	64 49	115 11	0 46	161
6 3/4	27 55	1 8	61 51	118 9	0 52	167
6 5/8	27 12	1 10	58 49	121 11	0 58	174
6 1/2	26 24	1 12	55 42	124 18	1 5	182
6 3/8	25 29	1 15	52 29	127 31	1 13	192
6 1/4	24 26	1 19	49 9	130 51	1 23	204
6 1/8	23 15	1 23	45 40	134 20	1 33	218
6	21 54	1 29	42 1	137 59	1 45	237
5 7/8	20 21	1 35	38 7	141 53	2 0	263
5 3/4	18 31	1 45	33 53	146 7	2 20	299
5 5/8	16 20	1 58	29 12	150 48	2 45	355
5 1/2	13 36	2 20	23 45	156 15	3 26	463
5 3/8	9 53	3 2	16 52	163 8	4 41	926
5 1/4	2 4	6 33	3 27	176 33	10 35	242
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 59°.

The Inclin to the Equat. is = 32°			The Latitude of Vertex is = 32°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	32 0	1 0	90		0 0	135
7⅞	31 58	1 0	87 21	92 39	0 4	136
7¾	31 53	1 1	84 41	95 19	0 9	136
7⅝	31 45	1 1	82 1	97 59	0 13	137
7½	31 33	1 1	79 20	100 40	0 18	139
7⅜	31 18	1 2	76 39	103 21	0 22	141
7¼	30 59	1 2	73 56	106 4	0 26	143
7⅛	30 36	1 3	71 11	108 49	0 31	147
7	30 9	1 5	68 24	111 36	0 37	150
6⅞	29 38	1 6	65 35	114 25	0 42	154
6¾	29 3	1 7	62 43	117 17	0 47	159
6⅝	28 22	1 9	59 47	120 13	0 54	166
6½	27 36	1 11	56 47	123 13	1 1	172
6⅜	26 44	1 14	53 42	126 18	1 8	181
6¼	25 45	1 17	50 32	129 28	1 15	192
6⅛	24 38	1 21	47 13	132 47	1 25	203
6	23 23	1 25	43 46	136 14	1 36	220
5⅞	21 56	1 31	40 7	139 53	1 48	241
5¾	20 16	1 39	36 13	143 47	2 4	268
5⅝	18 18	1 50	31 57	148 3	2 25	308
5½	15 56	2 5	27 11	152 49	2 52	374
5⅜	12 55	2 32	21 33	158 27	3 38	513
5¼	8 37	3 29	14 2	165 58	5 15	985
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 58°.

The Inclin. to the Equat. is = 33°			The Latitude of Vertex is = 33°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	33 0	1 0	90		0 0	130
7⅞	32 58	1 0	87 25	92 35	0 4	130
7¾	32 54	1 0	84 50	95 10	0 8	131
7⅝	32 46	1 0	82 14	97 46	0 12	132
7½	32 34	1 1	79 38	100 22	0 16	134
7⅜	32 20	1 1	77 1	102 59	0 20	135
7¼	32 1	1 3	74 22	105 38	0 25	138
7⅛	31 39	1 4	71 42	108 18	0 30	140
7	31 14	1 4	69 1	110 59	0 34	144
6⅞	30 44	1 5	66 17	113 43	0 39	147
6¾	30 10	1 7	63 30	116 30	0 45	152
6⅝	29 31	1 9	60 41	119 19	0 50	158
6½	28 47	1 11	57 48	122 12	0 56	164

Continuation of the Great Circle No. 33.						
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
6⅜	27 58	1 13	54 50	125 10	1 3	171
6¼	27 2	1 16	51 47	128 13	1 10	181
6⅛	25 59	1 19	48 38	131 22	1 18	191
6	24 48	1 23	45 22	134 38	1 27	205
5⅞	23 27	1 29	41 55	138 5	1 38	222
5¾	21 55	1 35	38 17	141 43	1 51	244
5⅝	20 8	1 43	34 22	145 38	2 7	275
5½	18 1	1 56	30 3	149 57	2 30	320
5⅜	15 27	2 13	25 11	154 49	3 0	398
5¼	12 6	2 46	19 17	160 43	3 53	589
5⅛	6 56	4 10	10 48	169 12	6 7	769
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 57°.

The Inclin. to the Equat. is = 34°			The Latitude of Vertex is = 34°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	34 0	1 0	90		0 0	125
7⅞	33 58	1 1	87 29	92 31	0 4	126
7¾	33 54	1 0	84 58	95 2	0 8	126
7⅝	33 46	1 1	82 26	97 34	0 12	127
7½	33 35	1 1	79 54	100 6	0 16	128
7⅜	33 21	1 2	77 21	102 39	0 20	130
7¼	33 4	1 2	74 47	105 13	0 24	132
7⅛	32 43	1 2	72 12	107 48	0 28	135
7	32 18	1 4	69 35	110 25	0 32	138
6⅞	31 49	1 5	66 56	113 4	0 37	141
6¾	31 17	1 6	64 15	115 45	0 41	145
6⅝	30 40	1 8	61 31	118 29	0 46	151
6½	29 58	1 10	58 44	121 16	0 52	156
6⅜	29 11	1 12	55 53	124 7	0 58	162
6¼	28 18	1 14	52 57	127 3	1 5	171
6⅛	27 18	1 18	49 56	130 4	1 12	180
6	26 11	1 22	46 49	133 11	1 20	192
5⅞	24 56	1 26	43 33	136 27	1 30	206
5¾	23 30	1 31	40 8	139 52	1 40	225
5⅝	21 51	1 39	36 29	143 31	1 54	249
5½	19 57	1 48	32 33	147 27	2 11	283
5⅜	17 40	2 2	28 11	151 49	2 34	334
5¼	14 52	2 23	23 10	156 50	3 9	429
5⅛	11 6	3 4	16 55	163 5	4 13	738
5	4 23	5 29	6 32	173 28	7 52	471
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 56°.

The Inclin. to the Equat is = 35° [illegible] The Latitude of Vertex is = 35°

Course	Lat.		diff.		Long. from the Intrs.		&		diff.		Dist.
Points	°	′	°	′	°	′	°	′	°	′	Miles
8	35	0	1	0	90°				0	0	
7 7/8	34	59	1	0	87	33	92	27	0	3	120
7 3/4	34	54	1	0	85	6	94	54	0	7	121
7 5/8	34	47	1	0	82	38	97	22	0	11	122
7 1/2	34	36	1	1	80	10	99	50	0	14	122
7 3/8	34	23	1	1	77	41	102	19	0	18	124
7 1/4	34	6	1	2	75	11	104	49	0	22	125
7 1/8	33	45	1	3	72	40	107	20	0	25	127
7	33	22	1	4	70	7	109	53	0	30	129
6 7/8	32	54	1	5	67	33	112	27	0	34	132
6 3/4	32	23	1	6	64	56	115	4	0	39	136
6 5/8	31	48	1	7	62	17	117	43	0	44	139
6 1/2	31	8	1	9	59	36	120	24	0	48	143
6 3/8	30	23	1	11	56	51	123	9	0	54	149
6 1/4	29	32	1	14	54	2	125	58	1	0	155
6 1/8	28	36	1	16	51	8	128	52	1	7	162
6	27	33	1	20	48	9	131	51	1	14	170
5 7/8	26	22	1	23	45	3	134	57	1	22	181
5 3/4	25	1	1	29	41	48	138	12	1	32	192
5 5/8	23	30	1	35	38	23	141	37	1	43	209
5 1/2	21	45	1	43	34	44	145	16	1	57	228
5 3/8	19	42	1	54	30	45	149	15	2	16	254
5 1/4	17	15	2	10	26	19	153	41	2	41	292
5 1/8	14	10	2	35	21	8	158	52	3	20	352
5	9	52	3	29	14	24	165	36	4	39	472
*	0	0	0	0	0	0	180	0	0	0	1044

* The Course in the Inters. with the Equat. is equal to 55°.

The Inclin. to the Equat. is = 36° [illegible] The Latitude of Vertex is = 36°

Course	Lat.		diff.		Long. from the Intrs.		&		diff.		Dist.
Points	°	′	°	′	°	′	°	′	°	′	Miles
8	36	0	1	0	90				0	0	
7 7/8	35	59	1	0	87	36	92	24	0	4	116
7 3/4	35	54	1	1	85	13	94	47	0	6	117
7 5/8	35	47	1	1	82	49	97	11	0	10	117
7 1/2	35	37	1	1	80	24	99	36	0	14	117
7 3/8	35	24	1	1	77	59	102	1	0	17	119
7 1/4	35	8	1	2	75	33	104	27	0	20	121
7 1/8	34	48	1	3	73	5	106	55	0	25	122
7	34	26	1	3	70	37	109	23	0	28	124
6 7/8	33	59	1	5	68	7	111	53	0	32	127
6 3/4	33	29	1	6	65	35	114	25	0	36	130
6 5/8	32	55	1	7	63	1	116	59	0	41	133
6 1/2	32	17	1	9	60	24	119	36	0	46	138
6 3/8	31	34	1	11	57	45	122	15	0	50	142
6 1/4	30	46	1	13	55	2	124	58	0	56	147
6 1/8	29	52	1	16	52	15	127	45	1	1	155
6	28	53	1	18	49	23	130	37	1	8	161
5 7/8	27	45	1	22	46	25	133	35	1	15	170
5 3/4	26	30	1	26	43	20	136	40	1	24	181
5 5/8	25	5	1	31	40	6	139	54	1	34	194
5 1/2	23	28	1	38	36	41	143	19	1	45	211
5 3/8	21	36	1	47	33	1	146	59	2	0	232
5 1/4	19	25	1	59	29	0	151	0	2	19	261
5 1/8	16	45	2	17	24	28	155	32	2	47	304
5	13	21	2	48	19	3	160	57	3	33	374
4 7/8	8	18	4	4	11	35	168	25	5	19	534
*	0	0	0	0	0	0	180	0	0	0	853

* The Course in the Inters. with the Equat. is equal to 54°.

(I.)

DESCRIPTION OF THE TABLES.

The number at the middle of the top of every table, *the Number of the Table*, is also that of the great circle embraced by the table, also the same number which the track of the circle in *the Scale of Great Circles* on the accompanying *Blank Chart* has. It denotes *the Inclination of the Great Circle to the Equator* (measured by the *Lat. of Vertex* of the circle) in whole degrees.

Besides that inclination of the circle; of the values contained in the tables and exactly denominated at the top of the pages, the courses are also adopted, but the latitudes, longitudes and distances are found by calculation.

Let the inclination of the circle $= B$, the adopted courses $= A', A'', A''', \dots,$ the latitudes of the places in which those courses take place, respectively, $= b', b'', b''' \dots,$ the belonging longitudes $= a', a'', a'''$ and the distances, reckoned, like the longitudes, from the intersection of the circle with the Equator, $= d', d'', d''', \dots,$ then B is the common acute angle of a series of right-angled spherical triangles, in which $b', b'', b''' \dots$ are the legs

The Inclin. to the Equat. is = 37° — **The Latitude of Vertex is = 37°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	37 0	1 0	90		0 0	112
7 7/8	36 59	1 0	87 40	92 20	0 3	112
7 3/4	36 55	1 0	85 19	94 41	0 7	113
7 5/8	36 48	1 0	82 59	97 1	0 9	114
7 1/2	36 38	1 1	80 38	99 22	0 12	114
7 3/8	36 25	1 1	78 16	101 44	0 16	116
7 1/4	36 10	1 1	75 53	104 7	0 20	118
7 1/8	35 51	1 2	73 30	106 30	0 23	119
7	35 29	1 3	71 5	108 55	0 26	122
6 7/8	35 4	1 4	68 39	111 21	0 30	125
6 3/4	34 35	1 5	66 11	113 49	0 34	128
6 5/8	34 2	1 7	63 42	116 18	0 38	131
6 1/2	33 26	1 8	61 10	118 50	0 42	136
6 3/8	32 45	1 10	58 35	121 25	0 47	141
6 1/4	31 59	1 12	55 58	124 2	0 52	147
6 1/8	31 8	1 14	53 16	126 44	0 58	153
6	30 11	1 17	50 31	129 29	1 3	161
5 7/8	29 7	1 21	47 40	132 20	1 10	171
5 3/4	27 56	1 25	44 44	135 16	1 17	182
5 5/8	26 36	1 29	41 40	138 20	1 25	196
5 1/2	25 6	1 35	38 26	141 34	1 36	214
5 3/8	23 23	1 42	35 1	144 59	1 48	237
5 1/4	21 24	1 52	31 19	148 41	2 4	269
5 1/8	19 2	2 6	27 15	152 45	2 25	317
5	16 9	2 27	22 36	157 24	2 56	402
4 7/8	12 22	3 5	16 54	163 6	3 50	638
4 3/4	6 7	5 3	8 11	171 49	6 27	612
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 53°.

The Inclin. to the Equat. is = 38° — **The Latitude of Vertex is = 38°**

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Seem
8	38 0	1 0	90		0 0	108
7 7/8	37 59	1 0	87 43	92 17	0 3	108
7 3/4	37 55	1 0	85 26	94 34	0 6	109
7 5/8	37 48	1 0	83 8	96 52	0 9	110
7 1/2	37 39	1 0	80 50	99 10	0 12	110
7 3/8	37 26	1 2	78 32	101 28	0 15	112
7 1/4	37 11	1 2	76 13	103 47	0 18	113
7 1/8	36 53	1 3	73 53	106 7	0 21	115
7	36 32	1 4	71 31	108 29	0 25	117
6 7/8	36 8	1 4	69 9	110 51	0 28	120
6 3/4	35 40	1 5	66 45	113 15	0 32	123
6 5/8	35 9	1 7	64 20	115 40	0 35	126
6 1/2	34 34	1 8	61 52	118 8	0 40	130
6 3/8	33 55	1 9	59 22	120 38	0 44	134
6 1/4	33 11	1 11	56 50	123 10	0 48	140
6 1/8	32 22	1 14	54 14	125 46	0 53	146
6	31 28	1 16	51 34	128 26	0 59	153
5 7/8	30 28	1 19	48 50	131 10	1 5	161
5 3/4	29 21	1 22	46 1	133 59	1 11	172
5 5/8	28 5	1 27	43 5	136 55	1 19	183
5 1/2	26 41	1 32	40 2	139 58	1 27	199
5 3/8	25 5	1 38	36 49	143 11	1 38	217
5 1/4	23 16	1 46	33 23	146 37	1 50	243
5 1/8	21 8	1 58	29 40	150 20	2 7	278
5	18 36	2 14	25 32	154 28	2 29	334
4 7/8	15 27	2 39	20 44	159 16	3 4	439
4 3/4	11 10	3 28	14 38	165 22	4 11	937
4 5/8	1 40	8 0	2 8	177 52	10 1	163
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 52°.

opposite to B; A', A'', A''' ...., respectively, the third angles of these triangles; a', a'', a'''.... the legs opposite to these angles and d', d'', d''', ... the hypotenuses. For determining the values to be found, we have therefore by the known solution of such triangles,

for the latitudes: $\cos b' = \dfrac{\cos B}{\sin A'}$, &c. &c., or log. $\cos b'$ = log. $\cos B$ + log. cosec A', &c. &c.

for those longitudes: $\cos a' = \dfrac{\cos A'}{\sin B}$, &c. &c., or log. $\cos a'$ = log. $\cos A'$ + log. cosec B, &c. &c.

for those distances: $\cos d' = \dfrac{\cot A'}{\tan B}$, &c. &c., or log. $\cos d'$ = log. $\cot A'$ + log. $\cot B$, &c. &c.

The actual calculation of these values is made by us to a precision of one second. In order to check any error of calculation, we have next constructed two *Kinds* of Tables of Differences (taking the 2nd, the 3th, &c., differences, where that was necessary). One of such containing the differences of the likenamed magnitudes always of one and the same circle, but the other containing the differences of the likenamed magnitudes corresponding to one

The Inclin. to the Equat. is = 39°.					The Latitude of Vertex is = 39°.	
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Strich	° ′	° ′	° ′	& ° ′	° ′	Miles
8	39 0	1 0	90		0 0	
$7^7/_8$	38 59	1 0	87 46	92 14	0 3	104
$7^3/_4$	38 55	1 0	85 32	94 28	0 5	105
$7^5/_8$	38 48	1 1	83 17	96 43	0 9	105
$7^1/_2$	38 39	1 1	81 2	98 58	0 12	105
$7^3/_8$	38 28	1 1	78 47	101 13	0 14	107
$7^1/_4$	38 13	1 2	76 31	103 29	0 17	107
$7^1/_8$	37 56	1 2	74 14	105 46	0 21	109
7	37 36	1 3	71 56	108 4	0 24	111
$6^7/_8$	37 12	1 4	69 37	110 23	0 27	113
$6^3/_4$	36 45	1 5	67 17	112 43	0 30	115
$6^5/_8$	36 16	1 6	64 55	115 5	0 34	118
$6^1/_2$	35 42	1 7	62 32	117 28	0 37	121
$6^3/_8$	35 4	1 9	60 6	119 54	0 41	125
$6^1/_4$	34 22	1 11	57 38	122 22	0 45	128
$6^1/_8$	33 36	1 13	55 7	124 53	0 50	134
6	32 44	1 15	52 33	127 27	0 55	139
$5^7/_8$	31 47	1 18	49 55	130 5	1 0	145
$5^3/_4$	30 43	1 21	47 12	132 48	1 6	153
$5^5/_8$	29 32	1 25	44 24	135 36	1 13	162
$5^1/_2$	28 13	1 29	41 29	138 31	1 21	172
$5^3/_8$	26 43	1 35	38 27	141 33	1 29	186
$5^1/_4$	25 2	1 42	35 13	144 47	1 40	201
$5^1/_8$	23 6	1 51	31 47	148 13	1 53	222
5	20 50	2 3	28 1	151 59	2 11	249
$4^7/_8$	18 6	2 21	23 48	156 12	2 35	290
$4^3/_4$	14 38	2 52	18 49	161 11	3 15	354
$4^5/_8$	9 40	4 0	12 9	167 51	4 41	491
*	0 0	0 0	0 0	180 0	0 0	929

* The Course in the Intrs. with the Equat. is equal to 51°.

The inclin. to the Equat. is = 40°.					The Latitude of Vertex is = 40°.	
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	40 0	1 0	90		0 0	
$7^7/_8$	39 59	1 0	87 49	92 11	0 2	101
$7^3/_4$	39 55	1 0	85 37	94 23	0 6	101
$7^5/_8$	39 49	1 0	83 26	96 34	0 8	101
$7^1/_2$	39 40	1 1	81 14	98 46	0 11	102
$7^3/_8$	39 29	1 1	79 1	100 59	0 14	102
$7^1/_4$	39 15	1 1	76 48	103 12	0 17	104
$7^1/_8$	38 58	1 2	74 35	105 25	0 19	105
7	38 39	1 2	72 20	107 40	0 22	107
$6^7/_8$	38 16	1 4	70 4	109 56	0 25	108
$6^3/_4$	37 50	1 5	67 47	112 13	0 29	111
$6^5/_8$	37 22	1 5	65 29	114 31	0 32	114
$6^1/_2$	36 49	1 7	63 9	116 51	0 35	116
$6^3/_8$	36 13	1 9	60 47	119 13	0 39	119
$6^1/_4$	35 33	1 10	58 23	121 37	0 43	123
$6^1/_8$	34 49	1 12	55 57	124 3	0 47	129
6	33 59	1 14	53 28	126 32	0 51	132
$5^7/_8$	33 5	1 16	50 55	129 5	0 56	138
$5^3/_4$	32 4	1 20	48 18	131 42	1 2	146
$5^5/_8$	30 57	1 23	45 37	134 23	1 7	153
$5^1/_2$	29 42	1 27	42 50	137 10	1 14	162
$5^3/_3$	28 18	1 32	39 56	140 4	1 22	174
$5^1/_4$	26 44	1 38	36 53	143 7	1 31	187
$5^1/_8$	24 57	1 46	33 40	146 20	1 42	205
5	22 53	1 56	30 12	149 48	1 56	227
$4^7/_8$	20 27	2 10	26 23	153 37	2 15	257
$4^3/_4$	17 30	2 31	22 4	157 56	2 42	303
$4^5/_8$	13 40	3 8	16 50	163 10	3 29	380
$4^1/_2$	7 42	4 48	9 16	170 44	5 30	571
*	0 0	0 0	0 0	180 0	0 0	722

* The Course in the Inters. with the Equat. is equal to 50°.

DESCRIPTION OF THE TABLES. (III.)

and the same Course of all the circles of the tables. After the verification of the calculation, every quantity of seconds less than 30 is left out and instead of 30, or more than 30 seconds, a whole minute is added. Exceptions of this principle are only made, if by applying it, the differences contained in the present tables give values, which disarrange the regularity of their increasing or decreasing. In these extraordinary cases, that principle is applied to the difference taken with keeping the seconds and then the chief number is formed by it.

Every one of the differences of latitudes and longitudes printed opposite to them is the difference of that dimension opposite to which it stands and the dimension of the same kind corresponding to the equal Course in the table of the following number.

Every distance contained in the column at the right side is the distance between the two places on the great circle which are determined by the latitudes and the longitudes of the two lines in the interval of which the distance is printed.

The Inclin. to the Equat. is = 41° — The Latitude of Vertex is = 41°

Course	Lat.		diff.		Long. from the Intrs.				diff.		Dist.
Points	°	′	°	′	°	′	& °	′	°	′	Miles
8	41	0	1	0	90				0	0	97
7 7/8	40	59	1	0	87	51	92	9	0	3	97
7 3/4	40	55	1	0	85	43	94	17	0	5	98
7 5/8	40	49	1	1	83	34	96	26	0	7	98
7 1/2	40	41	1	0	81	25	98	35	0	10	99
7 3/8	40	30	1	1	79	15	100	45	0	13	100
7 1/4	40	16	1	2	77	5	102	55	0	15	102
7 1/8	40	0	1	2	74	54	105	6	0	18	103
7	39	41	1	3	72	42	107	18	0	21	104
6 7/8	39	20	1	3	70	29	109	31	0	24	107
6 3/4	38	55	1	5	68	16	111	44	0	27	109
6 5/8	38	27	1	6	66	1	113	59	0	30	111
6 1/2	37	56	1	7	63	44	116	16	0	33	115
6 3/8	37	22	1	8	61	26	118	34	0	37	118
6 1/4	36	43	1	10	59	6	120	54	0	40	123
6 1/8	36	1	1	11	56	44	123	16	0	44	127
6	35	13	1	14	54	19	125	41	0	48	132
5 7/8	34	21	1	16	51	51	128	9	0	53	138
5 3/4	33	24	1	19	49	20	130	40	0	57	145
5 5/8	32	20	1	22	46	44	133	16	1	3	154
5 1/2	31	9	1	26	44	4	135	56	1	9	163
5 3/8	29	50	1	30	41	18	138	42	1	15	175
5 1/4	28	22	1	35	38	24	141	36	1	24	190
5 1/8	26	43	1	41	35	22	144	38	1	33	209
5	24	49	1	50	32	8	147	52	1	44	233
4 7/8	22	37	2	1	28	38	151	22	1	59	266
4 3/4	20	1	2	17	24	46	155	14	2	20	319
4 5/8	16	48	2	42	20	19	159	41	2	50	413
4 1/2	12	30	3	29	14	46	165	14	3	47	729
4 3/8	4	39	6	25	5	23	174	37	7	9	426
*	0	0	0	0	0	0	180	0	0	0	

* The Course in the Inters. with the Equat. is equal to 49°.

The Inclin. to the Equat. is = 42° — The Latitude of Vertex is = 42°

Course	Lat.		diff.		Long. from the Intrs.				diff.		Dist.
Points	°	′	°	′	°	′	& °	′	°	′	Miles
8	42	0	1	0	90				0	0	94
7 7/8	41	59	1	0	87	54	92	6	0	2	94
7 3/4	41	55	1	1	85	48	94	12	0	4	94
7 5/8	41	50	1	0	83	41	96	19	0	7	95
7 1/2	41	41	1	1	81	35	98	25	0	9	95
7 3/8	41	31	1	1	79	28	100	32	0	12	97
7 1/4	41	18	1	1	77	20	102	40	0	15	98
7 1/8	41	2	1	2	75	12	104	48	0	17	99
7	40	44	1	3	73	3	106	57	0	20	100
6 7/8	40	23	1	4	70	53	109	7	0	23	103
6 3/4	40	0	1	4	68	43	111	17	0	25	105
6 5/8	39	33	1	5	66	31	113	29	0	28	107
6 1/2	39	3	1	7	64	17	115	43	0	32	110
6 3/8	38	30	1	8	62	3	117	57	0	34	114
6 1/4	37	53	1	9	59	46	120	14	0	38	117
6 1/8	37	12	1	11	57	28	122	32	0	41	121
6	36	27	1	13	55	7	124	53	0	45	126
5 7/8	35	37	1	15	52	44	127	16	0	49	132
5 3/4	34	43	1	17	50	17	129	43	0	54	138
5 5/8	33	42	1	20	47	47	132	13	0	58	146
5 1/2	32	35	1	24	45	13	134	47	1	4	154
5 3/8	31	20	1	28	42	33	137	27	1	10	165
5 1/4	29	57	1	33	39	48	140	12	1	17	177
5 1/8	28	24	1	38	36	55	143	5	1	25	194
5	26	39	1	45	33	52	146	8	1	35	213
4 7/8	24	38	1	55	30	37	149	23	1	47	239
4 3/4	22	18	2	7	27	6	152	54	2	2	278
4 5/8	19	30	2	25	23	9	156	51	2	25	337
4 1/2	15	59	2	55	18	33	161	27	2	59	458
4 3/8	11	4	3	57	12	32	167	28	4	11	1000
*	0	0	0	0	0	0	180	0	0	0	

* The Course in the Inters. with the Equat. is equal to 48°.

DESCRIPTION OF THE TABLES. (IV.)

Any table is entered with the Course in the intersection of the circle with the Equator at the bottom and with the next Course to be expressed in eighths of the Point there at the left side.

The tables from No. 1 to No. 60 extend from the intersection of the circle with the Equator to its Vertex, but the tables from No. 61 to No. 89 embrace only arcs of the circles from the intersection with the Equator to latitudes somewhat greater then 60 degrees.

The attentive perusal of any table from its bottom to its top and, regarding the 2nd longitudes, backwards to the bottom, gives a clear and correct idea not only of a semicircle (arc of such) of that great circle *which is denoted* by the No. of the table,

The Inclin. to the Equat. is = 43°. The Latitude of Vertex is = 43°.

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	43 0	1 0	90		0 0	91
$7\frac{7}{8}$	42 59	1 0	87 56	92 4	0 3	90
$7\frac{3}{4}$	42 56	1 0	85 52	94 8	0 5	91
$7\frac{5}{8}$	42 50	1 0	83 48	96 12	0 7	92
$7\frac{1}{2}$	42 42	1 1	81 44	98 16	0 9	92
$7\frac{3}{8}$	42 32	1 1	79 40	100 20	0 11	93
$7\frac{1}{4}$	42 19	1 2	77 35	102 25	0 13	94
$7\frac{1}{8}$	42 4	1 2	75 29	104 31	0 16	96
7	41 47	1 3	73 23	106 37	0 18	97
$6\frac{7}{8}$	41 27	1 3	71 16	108 44	0 21	99
$6\frac{3}{4}$	41 4	1 4	69 8	110 52	0 24	101
$6\frac{5}{8}$	40 38	1 5	66 59	113 1	0 26	103
$6\frac{1}{2}$	40 10	1 6	64 49	115 11	0 29	106
$6\frac{3}{8}$	39 38	1 7	62 37	117 23	0 32	109
$6\frac{1}{4}$	39 2	1 9	60 24	119 36	0 35	112
$6\frac{1}{8}$	38 23	1 10	58 9	121 51	0 39	116
6	37 40	1 12	55 52	124 8	0 42	121
$5\frac{7}{8}$	36 52	1 14	53 33	126 27	0 46	126
$5\frac{3}{4}$	36 0	1 16	51 11	128 49	0 50	131
$5\frac{5}{8}$	35 2	1 19	48 45	131 15	0 55	138
$5\frac{1}{2}$	33 59	1 22	46 17	133 43	0 59	146
$5\frac{3}{8}$	32 48	1 26	43 43	136 17	1 5	156
$5\frac{1}{4}$	31 30	1 30	41 5	138 55	1 11	166
$5\frac{1}{8}$	30 2	1 36	38 20	141 40	1 18	180
5	28 24	1 42	35 27	144 33	1 26	197
$4\frac{7}{8}$	26 33	1 50	32 24	147 36	1 37	218
$4\frac{3}{4}$	24 25	2 0	29 8	150 52	1 50	248
$4\frac{5}{8}$	21 55	2 14	25 34	154 26	2 6	290
$4\frac{1}{2}$	18 54	2 35	21 32	158 28	2 31	361
$4\frac{3}{8}$	15 1	3 11	16 43	163 17	3 11	524
$4\frac{1}{4}$	9 14	4 38	10 2	169 58	4 47	816
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 47°.

The Inclin. to the Equat. is = 44°. The Latitude of Vertex is = 44°.

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	44 0	1 0	90		0 0	87
$7\frac{7}{8}$	43 59	1 0	87 59	92 1	0 2	88
$7\frac{3}{4}$	43 56	1 0	85 57	94 3	0 4	88
$7\frac{5}{8}$	43 50	1 1	83 55	96 5	0 7	88
$7\frac{1}{2}$	43 43	1 0	81 53	98 7	0 9	89
$7\frac{3}{8}$	43 33	1 1	79 51	100 9	0 11	90
$7\frac{1}{4}$	43 21	1 1	77 48	102 12	0 13	91
$7\frac{1}{8}$	43 6	1 2	75 45	104 15	0 15	92
7	42 50	1 2	73 41	106 19	0 18	94
$6\frac{7}{8}$	42 30	1 3	71 37	108 23	0 20	95
$6\frac{3}{4}$	42 8	1 4	69 32	110 28	0 22	97
$6\frac{5}{8}$	41 43	1 5	67 25	112 35	0 25	99
$6\frac{1}{2}$	41 16	1 6	65 18	114 42	0 28	102
$6\frac{3}{8}$	40 45	1 7	63 9	116 51	0 31	105
$6\frac{1}{4}$	40 11	1 8	60 59	119 1	0 34	108
$6\frac{1}{8}$	39 33	1 10	58 48	121 12	0 36	111
6	38 52	1 12	56 34	123 26	0 40	115
$5\frac{7}{8}$	38 6	1 14	54 19	125 41	0 43	121
$5\frac{3}{4}$	37 16	1 16	52 1	127 59	0 47	125
$5\frac{5}{8}$	36 21	1 19	49 40	130 20	0 51	131
$5\frac{1}{2}$	35 21	1 21	47 16	132 44	0 55	139
$5\frac{3}{8}$	34 14	1 24	44 48	135 12	1 0	147
$5\frac{1}{4}$	33 0	1 28	42 16	137 44	1 6	156
$5\frac{1}{8}$	31 38	1 33	39 38	140 22	1 12	169
5	30 6	1 38	36 53	143 7	1 20	183
$4\frac{7}{8}$	28 23	1 45	34 1	145 59	1 28	201
$4\frac{3}{4}$	26 25	1 54	30 58	149 2	1 38	224
$4\frac{5}{8}$	24 9	2 5	27 40	152 20	1 52	275
$4\frac{1}{2}$	21 29	2 21	24 3	155 57	2 10	305
$4\frac{3}{8}$	18 12	2 46	19 54	160 6	2 37	392
$4\frac{1}{4}$	13 52	3 31	14 49	165 11	3 26	633
$4\frac{1}{8}$	6 40	5 49	6 37	173 23	6 11	578
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 46°.

DESCRIPTION OF THE TABLES. (V.)

but also of the circle (or arc of it) of the succeeding number and besides of all great circles which are omitted between these two, considering sufficently the differences of latitudes and longitudes which the tables contain. These differences themselves namely determine the circle of the next higher number, any $\frac{1}{60}$ of them (of any of their degrees one minute, of any of their minutes one second) determines that great circle of which the inclination to the Equator (Lat. of Vertex) is one minute larger than that of the preceding circle.

For the use of the tables, the latitudes and geographical longitudes (with them Diff. Long.) of the two extremities of the ship's track are, first of all, supposed to be given; next the geographical longitude of the intersection of the Equator with the

The Inclin. to the Equat. is = 45°. The Latitude of Vertex is = 45°.

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	45 0	1 0	90		0 0	84
$7^7/_8$	44 59	1 0	88 1	91 59	0 2	85
$7^3/_4$	44 56	1 0	86 1	93 59	0 4	85
$7^5/_8$	44 51	1 0	84 2	95 58	0 6	85
$7^1/_2$	44 43	1 1	82 2	97 58	0 8	86
$7^3/_8$	44 34	1 1	80 2	99 58	0 10	87
$7^1/_4$	44 22	1 1	78 1	101 59	0 13	88
$7^1/_8$	44 8	1 2	76 0	104 0	0 15	88
7	43 52	1 2	73 59	106 1	0 17	91
$6^7/_8$	43 33	1 3	71 57	108 3	0 19	91
$6^3/_4$	43 12	1 4	69 54	110 6	0 21	94
$6^5/_8$	42 48	1 5	67 50	112 10	0 24	96
$6^1/_2$	42 22	1 5	65 46	114 14	0 26	97
$6^3/_8$	41 52	1 7	63 40	116 20	0 29	101
$6^1/_4$	41 19	1 8	61 33	118 27	0 32	103
$6^1/_8$	40 43	1 10	59 24	120 36	0 35	107
6	40 4	1 11	57 14	122 46	0 38	111
$5^7/_8$	39 20	1 13	55 2	124 58	0 41	115
$5^3/_4$	38 32	1 15	52 48	127 12	0 44	120
$5^5/_8$	37 40	1 17	50 31	129 29	0 48	125
$5^1/_2$	36 42	1 20	48 11	131 49	0 52	131
$5^3/_8$	35 38	1 23	45 48	134 12	0 57	140
$5^1/_4$	34 28	1 27	43 22	136 38	1 1	147
$5^1/_8$	33 11	1 31	40 50	139 10	1 7	159
5	31 44	1 36	38 13	141 47	1 13	170
$4^7/_8$	30 8	1 42	35 29	144 31	1 21	186
$4^3/_4$	28 19	1 49	32 36	147 24	1 30	206
$4^5/_8$	26 14	1 59	29 32	150 28	1 41	231
$4^1/_2$	23 50	2 11	26 13	153 47	1 55	268
$4^3/_8$	20 58	2 29	22 31	157 29	2 15	323
$4^1/_4$	17 23	2 59	18 15	161 45	2 45	431
$4^1/_8$	12 29	3 57	12 48	167 12	3 45	1069
4	0 0	0 0	0 0	180 0	0 0	

The Inclin. to the Equat. is = 46°. The Latitude of Vertex is = 46°.

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	46 0	1 0	90		0 0	81
$7^7/_8$	45 59	1 0	88 3	91 57	0 2	82
$7^3/_4$	45 56	1 0	86 5	93 55	0 4	82
$7^5/_8$	45 51	1 0	84 8	95 52	0 6	82
$7^1/_2$	45 44	1 0	82 10	97 50	0 8	83
$7^3/_8$	45 35	1 1	80 12	99 48	0 10	84
$7^1/_4$	45 23	1 2	78 14	101 46	0 12	85
$7^1/_8$	45 10	1 2	76 15	103 45	0 14	85
7	44 54	1 3	74 16	105 44	0 16	87
$6^7/_8$	44 36	1 3	72 16	107 44	0 18	89
$6^3/_4$	44 16	1 4	70 15	109 45	0 21	90
$6^5/_8$	43 53	1 4	68 14	111 46	0 23	92
$6^1/_2$	43 27	1 6	66 12	113 48	0 25	94
$6^3/_8$	42 59	1 6	64 9	115 51	0 27	97
$6^1/_4$	42 27	1 8	62 5	117 55	0 29	99
$6^1/_8$	41 53	1 9	59 59	120 1	0 32	103
6	41 15	1 10	57 52	122 8	0 35	106
$5^7/_8$	40 33	1 12	55 43	124 17	0 38	110
$5^3/_4$	39 47	1 14	53 32	126 28	0 42	114
$5^5/_8$	38 57	1 16	51 19	128 41	0 45	120
$5^1/_2$	38 2	1 19	49 3	130 57	0 49	125
$5^3/_8$	37 1	1 22	46 45	133 15	0 53	132
$5^1/_4$	35 55	1 25	44 23	135 37	0 57	140
$5^1/_8$	34 42	1 28	41 57	138 3	1 2	149
5	33 20	1 33	39 26	140 34	1 8	160
$4^7/_8$	31 50	1 38	36 50	143 10	1 14	174
$4^3/_4$	30 8	1 45	34 6	145 54	1 22	189
$4^5/_8$	28 13	1 53	31 13	148 47	1 31	211
$4^1/_2$	26 1	2 4	28 8	151 52	1 42	240
$4^3/_8$	23 27	2 18	24 46	155 14	1 58	279
$4^1/_4$	20 22	2 39	21 0	159 0	2 20	346
$4^1/_8$	16 26	3 14	16 33	163 27	2 55	487
4	10 46	4 33	10 35	169 25	4 13	903
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 44°.

DESCRIPTION OF THE TABLES. (VI.)

great circle passing through those two places is to be determined, as is the inclination of that circle to the Equator.

If this inclination of the circle (Lat. of Vertex) contains only whole degrees, the table the No. of which is equal to that quantity of degrees shews directly the latitudes to be applied (the latitudes contained in the table lying between the given latitudes, if we consider Diff. Long.); farther, that table shews directly the spherical courses and the distances corresponding to those latitudes; whereas the respective

The Inclin. to the Equat. is = 47°.			The Latitude of Vertex is = 47°.			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	47 0	1 0	90		0 0	
$7^{7}/_{8}$	46 59	1 0	88 5	91 55	0 1	79
$7^{3}/_{4}$	46 56	1 0	86 9	93 51	0 4	79
$7^{5}/_{8}$	46 51	1 1	84 14	95 46	0 5	79
$7^{1}/_{2}$	46 44	1 1	82 18	97 42	0 7	79
$7^{3}/_{8}$	46 36	1 0	80 22	99 38	0 9	80
$7^{1}/_{4}$	46 25	1 1	78 26	101 34	0 11	81
$7^{1}/_{8}$	46 12	1 1	76 29	103 31	0 13	82
7	45 57	1 2	74 32	105 28	0 15	82
$6^{7}/_{8}$	45 39	1 3	72 34	107 26	0 17	84
$6^{3}/_{4}$	45 20	1 3	70 36	109 24	0 19	85
$6^{5}/_{8}$	44 57	1 5	68 37	111 23	0 21	87
$6^{1}/_{2}$	44 33	1 5	66 37	113 23	0 23	89
$6^{3}/_{8}$	44 5	1 7	64 36	115 24	0 26	91
$6^{1}/_{4}$	43 35	1 8	62 34	117 26	0 28	93
$6^{1}/_{8}$	43 2	1 9	60 31	119 29	0 31	95
6	42 25	1 10	58 27	121 33	0 33	98
$5^{7}/_{8}$	41 45	1 12	56 21	123 39	0 36	102
$5^{3}/_{4}$	41 1	1 14	54 14	125 46	0 39	105
$5^{5}/_{8}$	40 13	1 16	52 4	127 56	0 42	110
$5^{1}/_{2}$	39 21	1 18	49 52	130 8	0 46	114
$5^{3}/_{8}$	38 23	1 21	47 38	132 22	0 49	119
$5^{1}/_{4}$	37 20	1 24	45 20	134 40	0 54	126
$5^{1}/_{8}$	36 10	1 28	42 59	137 1	0 58	133
5	34 53	1 32	40 34	139 26	1 3	141
$4^{7}/_{8}$	33 28	1 36	38 4	141 56	1 9	150
$4^{3}/_{4}$	31 53	1 42	35 28	144 32	1 15	162
$4^{5}/_{8}$	30 6	1 49	32 44	147 16	1 23	177
$4^{1}/_{2}$	28 5	1 58	29 50	150 10	1 33	194
$4^{3}/_{8}$	25 54	2 10	26 44	153 16	1 45	217
$4^{1}/_{4}$	23 1	2 25	23 20	156 40	2 1	248
$4^{1}/_{8}$	19 40	2 50	19 28	160 32	2 26	295
4	15 19	3 33	14 48	165 12	3 7	374
$3^{7}/_{8}$	8 29	5 30	8 0	172 0	4 57	572
*	0 0	0 0	0 0	180 0	0 0	698

* The Course in the Inters. with the Equat. is equal to 43°.

The Inclin. to the Equat. is = 48°.			The Latitude of Vertex is = 48°.			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′	& ° ′	° ′	Miles
8	48 0	1 0	90		0 0	
$7^{7}/_{8}$	47 59	1 0	88 6	91 54	0 2	76
$7^{3}/_{4}$	47 56	1 0	86 13	93 47	0 3	76
$7^{5}/_{8}$	47 52	1 0	84 19	95 41	0 5	76
$7^{1}/_{2}$	47 45	1 0	82 25	97 35	0 7	77
$7^{3}/_{8}$	47 36	1 1	80 31	99 29	0 9	78
$7^{1}/_{4}$	47 26	1 1	78 37	101 23	0 10	78
$7^{1}/_{8}$	47 13	1 2	76 42	103 18	0 12	78
7	46 59	1 2	74 47	105 13	0 14	80
$6^{7}/_{8}$	46 42	1 3	72 51	107 9	0 16	81
$6^{3}/_{4}$	46 23	1 4	70 55	109 5	0 18	82
$6^{5}/_{8}$	46 2	1 4	68 58	111 2	0 20	84
$6^{1}/_{2}$	45 38	1 5	67 0	113 0	0 23	85
$6^{3}/_{8}$	45 12	1 6	65 2	114 58	0 25	87
$6^{1}/_{4}$	44 43	1 7	63 2	116 58	0 27	90
$6^{1}/_{8}$	44 11	1 8	61 2	118 58	0 29	91
6	43 35	1 10	59 0	121 0	0 32	95
$5^{7}/_{8}$	42 57	1 11	56 57	123 3	0 35	97
$5^{3}/_{4}$	42 15	1 13	54 53	125 7	0 37	101
$5^{5}/_{8}$	41 29	1 15	52 46	127 14	0 40	105
$5^{1}/_{2}$	40 39	1 17	50 38	129 22	0 43	109
$5^{3}/_{8}$	39 44	1 19	48 27	131 33	0 46	114
$5^{1}/_{4}$	38 44	1 22	46 14	133 46	0 50	120
$5^{1}/_{8}$	37 38	1 25	43 57	136 3	0 54	126
5	36 25	1 29	41 37	138 23	0 59	133
$4^{7}/_{8}$	35 4	1 34	39 13	140 47	1 4	142
$4^{3}/_{4}$	33 35	1 39	36 43	143 17	1 10	153
$4^{5}/_{8}$	31 55	1 46	34 7	145 53	1 17	165
$4^{1}/_{2}$	30 3	1 53	31 23	148 37	1 25	180
$4^{3}/_{8}$	27 55	2 2	28 29	151 31	1 35	199
$4^{1}/_{4}$	25 26	2 16	25 21	154 39	1 48	224
$4^{1}/_{8}$	22 30	2 34	21 54	158 6	2 5	259
4	18 52	3 2	17 55	162 5	2 33	312
$3^{7}/_{8}$	13 59	3 57	12 57	167 3	3 23	409
$3^{3}/_{4}$	4 52	7 28	4 24	175 36	6 33	744
*	0 0	0 0	0 0	180 0	0 0	394

* The Course in the Inters. with the Equat. is equal to 42°.

DESCRIPTION OF THE TABLES. (VII.)

longitudes must be reduced, before their application, to the geographical longitudes in which the spherical Courses supposed in the table take place. But this reduction is simply done by adding algebraically the geographical longitude of the intersection if the great circle with the Equator to any one of those longitudes of the table.

If the inclination of the great circle to the Equator (Lat. of Vertex) contains

The Inclin. to the Equat. is = 49°. [redacted] The Latitude of Vertex is = 49°.						
Course	Lat.	diff.	Long. from the Intr.		diff.	Dist.
Points	° ′	° ′	° ′ 90 &	° ′	° ′	Miles
8	49 0	1 0	90		0 0	
7⅞	48 59	1 0	88 8	91 52	0 2	73
7¾	48 56	1 1	86 16	93 44	0 4	74
7⅝	48 52	1 0	84 24	95 36	0 5	74
7½	48 46	1 0	82 32	97 28	0 7	74
7⅜	48 37	1 1	80 40	99 20	0 8	74
7¼	48 27	1 1	78 47	101 13	0 10	76
7⅛	48 15	1 2	76 54	103 6	0 12	76
7	48 1	1 2	75 1	104 59	0 14	76
6⅞	47 45	1 3	73 7	106 53	0 16	78
6¾	47 27	1 3	71 13	108 47	0 17	80
6⅝	47 6	1 4	69 18	110 42	0 19	80
6½	46 43	1 5	67 23	112 37	0 21	82
6⅜	46 18	1 6	65 27	114 33	0 23	84
6¼	45 50	1 7	63 29	116 31	0 26	86
6⅛	45 19	1 8	61 31	118 29	0 28	88
6	44 45	1 10	59 32	120 28	0 30	91
5⅞	44 8	1 11	57 32	122 28	0 32	94
5¾	43 28	1 13	55 30	124 30	0 34	97
5⅝	42 44	1 15	53 26	126 34	0 38	100
5½	41 56	1 17	51 21	128 39	0 40	104
5⅜	41 3	1 19	49 13	130 47	0 44	109
5¼	40 6	1 22	47 4	132 56	0 47	114
5⅛	39 3	1 25	44 51	135 9	0 51	120
5	37 54	1 28	42 36	137 24	0 55	127
4⅞	36 38	1 32	40 17	139 43	0 59	134
4¾	35 14	1 37	37 53	142 7	1 4	144
4⅝	33 41	1 42	35 24	144 36	1 10	155
4½	31 56	1 49	32 48	147 12	1 18	168
4⅜	29 57	1 58	30 4	149 56	1 26	184
4¼	27 42	2 8	27 9	152 51	1 37	204
4⅛	25 4	2 22	23 59	156 1	1 50	232
4	21 54	2 44	20 28	159 32	2 9	272
3⅞	17 56	3 17	16 20	163 40	2 41	333
3¾	12 20	4 30	10 57	169 3	3 45	458
*	0 0	0 0	0 0	180 0	0 0	986

* The Course in the Inters. with the Equat. is equal to 41°.

The Inclin. to the Equat. is = 50°. [redacted] The Latitude of Vertex is = 50°.						
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ 90 &	° ′	° ′	Miles
8	50 0	1 0	90		0 0	
7⅞	49 59	1 0	88 10	91 50	0 1	71
7¾	49 57	1 0	86 20	93 40	0 3	71
7⅝	49 52	1 0	84 29	95 31	0 5	71
7½	49 46	1 1	82 39	97 21	0 6	71
7⅜	49 38	1 1	80 48	99 12	0 8	72
7¼	49 28	1 1	78 57	101 3	0 10	73
7⅛	49 17	1 1	77 6	102 54	0 12	73
7	49 3	1 2	75 15	104 45	0 13	74
6⅞	48 48	1 2	73 23	106 37	0 14	76
6¾	48 30	1 3	71 30	108 30	0 17	76
6⅝	48 10	1 4	69 37	110 23	0 19	78
6½	47 48	1 5	67 44	112 16	0 20	79
6⅜	47 24	1 5	65 50	114 10	0 22	81
6¼	46 57	1 6	63 55	116 5	0 24	82
6⅛	46 27	1 8	61 59	118 1	0 26	85
6	45 55	1 9	60 2	119 58	0 28	87
5⅞	45 19	1 11	58 4	121 56	0 30	90
5¾	44 41	1 12	56 4	123 56	0 33	93
5⅝	43 59	1 13	54 4	125 56	0 35	96
5½	43 13	1 15	52 1	127 59	0 38	100
5⅜	42 22	1 18	49 57	130 3	0 41	104
5¼	41 28	1 20	47 51	132 9	0 44	109
5⅛	40 28	1 23	45 42	134 18	0 48	114
5	39 22	1 27	43 31	136 29	0 51	120
4⅞	38 10	1 30	41 16	138 44	0 55	127
4¾	36 51	1 34	38 57	141 3	1 0	136
4⅝	35 23	1 39	36 34	143 26	1 5	145
4½	33 45	1 45	34 6	145 54	1 11	157
4⅜	31 55	1 52	31 30	148 30	1 19	171
4¼	29 50	2 2	28 46	151 14	1 27	188
4⅛	27 26	2 14	25 49	154 11	1 39	211
4	24 38	2 30	22 37	157 23	1 54	242
3⅞	21 13	2 54	19 1	160 59	2 15	285
3¾	16 50	3 36	14 42	165 18	2 51	359
3⅝	10 14	5 18	8 43	171 17	4 17	529
*	0 0	0 0	0 0	180 0	0 0	804

* The Course in the Inters. with the Equat. is equal to 40°.

DESCRIPTION OF THE TABLES. (VIII.)

besides whole degrees also minutes, that table the No. of which is equal to this quantity of whole degrees is to be made use of. But before the latitudes and longitudes of this table are to be farther applied, to any one of them is to be added the product of one sixtieth of the belonging difference (of any degree of the difference one minute, of any minute of it one second) by the quantity of minutes of the circle's

The Inclin. to the Equat. is = 51° [illegible] The Latitude of Vertex is = 51°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° '	° '	° '	& ° '	° '	Miles
8	51 0	1 0	90		0 0	68
7 7/8	50 59	1 0	88 11	91 49	0 2	69
7 3/4	50 57	1 0	86 23	93 37	0 3	68
7 5/8	50 52	1 1	84 34	95 26	0 5	69
7 1/2	50 47	1 0	82 45	97 15	0 6	70
7 3/8	50 39	1 1	80 56	99 4	0 8	70
7 1/4	50 29	1 2	79 7	100 53	0 9	71
7 1/8	50 18	1 2	77 18	102 42	0 10	71
7	50 5	1 2	75 28	104 32	0 12	73
6 7/8	49 50	1 3	73 37	106 23	0 14	73
6 3/4	49 33	1 3	71 47	108 13	0 15	75
6 5/8	49 14	1 4	69 56	110 4	0 17	76
6 1/2	48 53	1 4	68 4	111 56	0 19	78
6 3/8	48 29	1 6	66 12	113 48	0 20	80
6 1/4	48 3	1 7	64 19	115 41	0 22	81
6 1/8	47 35	1 8	62 25	117 35	0 24	84
6	47 4	1 9	60 30	119 30	0 27	86
5 7/8	46 30	1 10	58 34	121 26	0 29	89
5 3/4	45 53	1 12	56 37	123 23	0 31	92
5 5/8	45 12	1 14	54 39	125 21	0 33	96
5 1/2	44 28	1 16	52 39	127 21	0 36	99
5 3/8	43 40	1 18	50 38	129 22	0 39	104
5 1/4	42 48	1 20	48 35	131 25	0 42	109
5 1/8	41 51	1 22	46 30	133 30	0 44	114
5	40 49	1 25	44 22	135 38	0 48	121
4 7/8	39 40	1 29	42 11	137 49	0 52	129
4 3/4	38 25	1 32	39 57	140 3	0 56	137
4 5/8	37 2	1 37	37 39	142 21	1 1	147
4 1/2	35 30	1 42	35 17	144 43	1 6	160
4 3/8	33 47	1 49	32 49	147 11	1 12	174
4 1/4	31 52	1 56	30 13	149 47	1 20	194
4 1/8	29 40	2 7	27 28	152 32	1 29	217
4	27 8	2 20	24 31	155 29	1 40	252
3 7/8	24 7	2 39	21 16	158 44	1 56	303
3 3/4	20 26	3 7	17 33	162 27	2 21	392
3 5/8	15 32	3 59	13 0	167 0	3 4	648
3 1/2	7 15	6 42	5 55	174 5	5 17	561
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 39°.

The Inclin. to the Equat. is = 52° [illegible] The Latitude of Vertex is = 52°

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° '	° '	° '	& ° '	° '	Miles
8	52 0	1 0	90		0 0	66
7 7/8	51 59	1 0	88 13	91 47	0 1	66
7 3/4	51 57	1 0	86 26	93 34	0 3	66
7 5/8	51 53	1 0	84 39	95 21	0 4	67
7 1/2	51 47	1 0	82 51	97 9	0 6	67
7 3/8	51 40	1 0	81 4	98 56	0 7	67
7 1/4	51 31	1 1	79 16	100 44	0 9	68
7 1/8	51 20	1 1	77 28	102 32	0 10	69
7	51 7	1 2	75 40	104 20	0 12	70
6 7/8	50 53	1 2	73 51	106 9	0 14	71
6 3/4	50 36	1 3	72 2	107 58	0 15	72
6 5/8	50 18	1 4	70 13	109 47	0 16	74
6 1/2	49 57	1 5	68 23	111 37	0 18	74
6 3/8	49 35	1 5	66 32	113 28	0 20	77
6 1/4	49 10	1 6	64 41	115 19	0 22	78
6 1/8	48 43	1 7	62 49	117 11	0 24	81
6	48 13	1 8	60 57	119 3	0 25	83
5 7/8	47 40	1 10	59 3	120 57	0 27	85
5 3/4	47 5	1 11	57 8	122 52	0 30	88
5 5/8	46 26	1 13	55 12	124 48	0 32	92
5 1/2	45 44	1 14	53 15	126 45	0 35	95
5 3/8	44 58	1 16	51 17	128 43	0 36	99
5 1/4	44 8	1 19	49 17	130 43	0 39	104
5 1/8	43 13	1 22	47 14	132 46	0 42	109
5	42 14	1 24	45 10	134 50	0 45	115
4 7/8	41 9	1 27	43 3	136 57	0 49	122
4 3/4	39 57	1 31	40 53	139 7	0 53	129
4 5/8	38 39	1 35	38 40	141 20	0 57	139
4 1/2	37 12	1 40	36 23	143 37	1 1	149
4 3/8	35 36	1 46	34 1	145 59	1 7	163
4 1/4	33 48	1 53	31 33	148 27	1 13	179
4 1/8	31 47	2 1	28 57	151 3	1 21	199
4	29 28	2 12	26 11	153 49	1 31	226
3 7/8	26 46	2 27	23 12	156 48	1 44	264
3 3/4	23 33	2 48	19 54	160 6	2 1	322
3 5/8	19 31	3 21	16 4	163 56	2 28	436
3 1/2	13 57	4 30	11 12	168 48	3 21	1069
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 38°.

DESCRIPTION OF THE TABLES. (IX.)

inclination. The *distances of the Tables* are to be worked in the same manner; but before this, any one of the belonging differences is to be formed by the subtraction of the respective two distances which stand in the intervals of the lines of the same two Courses, the one in the table in use, the other in the table of the next higher No.

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
The Inclin. to the Equat. is = 53°.			The Latitude of Vertex is = 53°.			
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	53 0	1 0	90		0 0	64
7 7/8	52 59	1 0	88 14	91 46	0 2	63
7 3/4	52 57	1 0	86 29	93 31	0 2	64
7 5/8	52 53	1 0	84 43	95 17	0 4	64
7 1/2	52 47	1 1	82 57	97 3	0 6	65
7 3/8	52 40	1 1	81 11	98 49	0 7	65
7 1/4	52 32	1 1	79 25	100 35	0 8	66
7 1/8	52 21	1 2	77 38	102 22	0 10	66
7	52 9	1 2	75 52	104 8	0 11	68
6 7/8	51 55	1 2	74 5	105 55	0 12	68
6 3/4	51 39	1 3	72 17	107 43	0 14	69
6 5/8	51 22	1 3	70 29	109 31	0 16	71
6 1/2	51 2	1 4	68 41	111 19	0 17	72
6 3/8	50 40	1 5	66 52	113 8	0 19	74
6 1/4	50 16	1 6	65 3	114 57	0 20	75
6 1/8	49 50	1 7	63 13	116 47	0 22	77
6	49 21	1 8	61 22	118 38	0 24	80
5 7/8	48 50	1 9	59 30	120 30	0 26	82
5 3/4	48 16	1 11	57 38	122 22	0 28	84
5 5/8	47 39	1 12	55 44	124 16	0 30	88
5 1/2	46 58	1 14	53 50	126 10	0 32	91
5 3/8	46 14	1 16	51 53	128 7	0 35	95
5 1/4	45 27	1 18	49 56	130 4	0 37	99
5 1/8	44 35	1 20	47 56	132 4	0 40	104
5	43 38	1 23	45 55	134 5	0 43	109
4 7/8	42 36	1 26	43 52	136 8	0 45	116
4 3/4	41 28	1 30	41 46	138 14	1 49	122
4 5/8	40 14	1 33	39 37	140 23	1 53	131
4 1/2	38 52	1 38	37 24	142 36	1 57	141
4 3/8	37 22	1 43	35 8	144 52	1 2	152
4 1/4	35 41	1 49	32 46	147 14	1 8	166
4 1/8	33 48	1 57	30 18	149 42	1 14	183
4	31 40	2 6	27 42	152 18	1 22	205
3 7/8	29 13	2 18	24 56	155 4	1 32	236
3 3/4	26 21	2 34	21 55	158 5	1 45	278
3 5/8	22 52	2 59	18 32	161 28	2 5	347
3 1/2	18 27	3 39	14 33	165 27	2 37	496
3 3/8	11 59	5 12	9 12	170 48	3 47	904
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 37°.

Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
The Inclin. to the Equat. is = 54°.			The Latitude of Vertex is = 54°.			
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	54 0	1 0	90		0 0	61
7 7/8	53 59	1 0	88 16	91 44	0 1	62
7 3/4	53 57	1 0	86 31	93 29	0 3	61
7 5/8	53 53	1 0	84 47	95 13	0 4	62
7 1/2	53 48	1 0	83 3	96 57	0 5	62
7 3/8	53 41	1 1	81 18	98 42	0 6	63
7 1/4	53 33	1 1	79 33	100 27	0 8	64
7 1/8	53 23	1 1	77 48	102 12	0 9	64
7	53 11	1 2	76 3	103 57	0 10	65
6 7/8	52 57	1 3	74 17	105 43	0 12	65
6 3/4	52 42	1 3	72 31	107 29	0 14	67
6 5/8	52 25	1 4	70 45	109 15	0 15	68
6 1/2	52 6	1 5	68 58	111 2	0 17	69
6 3/8	51 45	1 5	67 11	112 49	0 18	71
6 1/4	51 22	1 6	65 23	114 37	0 20	73
6 1/8	50 57	1 7	63 35	116 25	0 21	74
6	50 29	1 8	61 46	118 14	0 23	76
5 7/8	49 59	1 9	59 56	120 4	0 25	79
5 3/4	49 27	1 10	58 6	121 54	0 26	81
5 5/8	48 51	1 12	56 14	123 46	0 29	84
5 1/2	48 12	1 14	54 22	125 38	0 30	87
5 3/8	47 30	1 16	52 28	127 32	0 32	91
5 1/4	46 45	1 17	50 33	129 27	0 35	95
5 1/8	45 55	1 19	48 36	131 24	0 37	99
5	45 1	1 22	46 38	133 22	0 40	104
4 7/8	44 2	1 25	44 37	135 23	0 43	109
4 3/4	42 58	1 28	42 35	137 25	0 46	116
4 5/8	41 47	1 32	40 30	139 30	0 49	124
4 1/2	40 30	1 36	38 21	141 39	0 53	132
4 3/8	39 5	1 40	36 10	143 50	0 57	143
4 1/4	37 30	1 47	33 54	146 6	1 2	155
4 1/8	35 45	1 53	31 32	148 28	1 8	170
4	33 46	2 1	29 4	150 56	1 15	188
3 7/8	31 31	2 12	26 28	153 32	1 23	213
3 3/4	28 55	2 25	23 40	156 20	1 34	246
3 5/8	25 51	2 44	20 37	159 23	1 48	294
3 1/2	22 6	3 12	17 10	162 50	2 9	378
3 3/8	17 11	4 1	12 59	167 1	2 47	590
3 1/4	9 21	6 19	6 52	173 8	4 27	695
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 36°.

The Inclin. to the Equat. is = 55°.						The Latitude of Vertex is = 55°.
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	55 0	1 0	90		0 0	
7 7/8	54 59	1 0	88 17	91 43	0 1	59
7 3/4	54 57	1 0	86 34	93 26	0 3	59
7 5/8	54 53	1 1	84 51	95 9	0 4	60
7 1/2	54 48	1 1	83 8	96 52	0 5	59
7 3/8	54 42	1 0	81 24	98 36	0 6	60
7 1/4	54 34	1 1	79 41	100 19	0 7	61
7 1/8	54 24	1 1	77 57	102 3	0 9	61
7	54 13	1 1	76 13	103 47	0 10	61
6 7/8	54 0	1 2	74 29	105 31	0 11	63
6 3/4	53 45	1 3	72 45	107 15	0 12	63
6 5/8	53 29	1 3	71 0	109 0	0 14	64
6 1/2	53 11	1 4	69 15	110 45	0 15	66
6 3/8	52 50	1 5	67 29	112 31	0 17	67
6 1/4	52 28	1 6	65 43	114 17	0 18	68
6 1/8	52 4	1 7	63 56	116 4	0 20	69
6	51 37	1 8	62 9	117 51	0 22	72
5 7/8	51 8	1 9	60 21	119 39	0 23	73
5 3/4	50 37	1 10	58 32	121 28	0 25	75
5 5/8	50 3	1 12	56 43	123 17	0 27	78
5 1/2	49 26	1 13	54 52	125 8	0 29	81
5 3/8	48 46	1 15	53 0	127 0	0 31	83
5 1/4	48 2	1 17	51 8	128 52	0 33	87
5 1/8	47 14	1 19	49 13	130 47	0 36	90
5	46 23	1 21	47 18	132 42	0 37	95
4 7/8	45 27	1 24	45 20	134 40	0 40	99
4 3/4	44 26	1 27	43 21	136 39	0 43	104
4 5/8	43 19	1 30	41 19	138 41	0 46	110
4 1/2	42 6	1 34	39 14	140 46	0 50	117
4 3/8	40 45	1 39	37 7	142 53	0 54	125
4 1/4	39 17	1 43	34 56	145 4	0 58	135
4 1/8	37 38	1 49	32 40	147 20	1 3	145
4	35 47	1 57	30 19	149 41	1 9	158
3 7/8	33 43	2 6	27 51	152 9	1 16	174
3 3/4	31 20	2 18	25 14	154 46	1 25	194
3 5/8	28 35	2 32	22 25	157 35	1 37	221
3 1/2	25 18	2 53	19 19	160 41	1 52	258
3 3/8	21 12	3 27	15 46	164 14	2 16	314
3 1/4	15 40	4 30	11 19	168 41	3 1	417
3 1/8	5 3	8 45	3 33	176 27	5 59	785
*	0 0	0 0	0 0	180 0	0 0	370

* The Course in the Inters. with the Equat. is equal to 35°.

The Inclin. to the Equat. is = 56°.						The Latitude of Vertex is = 56°.
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
8	56 0	1 0	90		0 0	
7 7/8	55 59	1 0	88 18	91 42	0 1	57
7 3/4	55 57	1 0	86 37	93 23	0 2	57
7 5/8	55 54	1 0	84 55	95 5	0 3	57
7 1/2	55 49	1 0	83 13	96 47	0 4	57
7 3/8	55 42	1 1	81 30	98 30	0 6	58
7 1/4	55 35	1 1	79 48	100 12	0 7	59
7 1/8	55 25	1 2	78 6	101 54	0 8	58
7	55 14	1 2	76 23	103 37	0 10	60
6 7/8	55 2	1 2	74 40	105 20	0 11	60
6 3/4	54 48	1 3	72 57	107 3	0 12	61
6 5/8	54 32	1 4	71 14	108 46	0 13	61
6 1/2	54 15	1 4	69 30	110 30	0 15	63
6 3/8	53 55	1 5	67 46	112 14	0 16	64
6 1/4	53 34	1 5	66 1	113 59	0 18	66
6 1/8	53 11	1 6	64 16	115 44	0 19	67
6	52 45	1 8	62 31	117 29	0 20	68
5 7/8	52 17	1 9	60 44	119 16	0 22	71
5 3/4	51 47	1 10	58 57	121 3	0 24	72
5 5/8	51 15	1 11	57 10	122 50	0 25	75
5 1/2	50 39	1 13	55 21	124 39	0 27	77
5 3/8	50 1	1 14	53 31	126 29	0 29	80
5 1/4	49 19	1 16	51 41	128 19	0 31	83
5 1/8	48 33	1 19	49 49	130 11	0 33	86
5	47 44	1 21	47 55	132 5	0 36	90
4 7/8	46 51	1 23	46 0	134 0	0 38	95
4 3/4	45 53	1 25	44 4	135 56	0 41	99
4 5/8	44 49	1 29	42 5	137 55	0 44	105
4 1/2	43 40	1 32	40 4	139 56	0 47	111
4 3/8	42 24	1 36	38 1	141 59	0 50	118
4 1/4	41 0	1 41	35 54	144 6	0 54	126
4 1/8	39 27	1 47	33 43	146 17	0 59	136
4	37 44	1 54	31 28	148 32	1 4	148
3 7/8	35 49	2 1	29 7	150 53	1 10	162
3 3/4	33 38	2 10	26 39	153 21	1 17	178
3 5/8	31 7	2 23	24 2	155 58	1 26	201
3 1/2	28 11	2 40	21 11	158 49	1 38	230
3 3/8	24 39	3 4	18 2	161 58	1 55	272
3 1/4	20 10	3 44	14 20	165 40	2 23	338
3 1/8	13 48	5 8	9 32	170 28	3 21	471
*	0 0	0 0	0 0	180 0	0 0	1003

* The Course in the Inters. with the Equat. is equal to 34°.

The Inclin. to the Equat. is = 57°.	[redacted]					The Latitude of Vertex is = 57°.					
Course	Lat.		diff.		Long. from the Intrs.				diff.		Dist.
Points	°	′	°	′	°	′ &	°	′	°	′	Miles
8	57	0	1	0	90				0	0	55
7⅞	56	59	1	0	88	19	91	41	0	2	55
7¾	56	57	1	0	86	39	93	21	0	2	55
7⅝	56	54	1	0	84	58	95	2	0	3	55
7½	56	49	1	1	83	17	96	43	0	5	56
7⅜	56	43	1	1	81	36	98	24	0	6	56
7¼	56	36	1	0	79	55	100	5	0	7	56
7⅛	56	27	1	1	78	14	101	46	0	8	57
7	56	16	1	2	76	33	103	27	0	9	58
6⅞	56	4	1	2	74	51	105	9	0	11	59
6¾	55	51	1	2	73	9	106	51	0	12	59
6⅝	55	36	1	3	71	27	108	33	0	13	61
6½	55	19	1	3	69	45	110	15	0	14	61
6⅜	55	0	1	5	68	2	111	58	0	15	63
6¼	54	59	1	6	66	19	113	41	0	17	64
6⅛	54	17	1	6	64	35	115	25	0	18	66
6	53	53	1	7	62	51	117	9	0	20	68
5⅞	53	26	1	8	61	6	118	54	0	21	69
5¾	52	57	1	10	59	21	120	39	0	22	72
5⅝	52	26	1	11	57	35	122	25	0	24	74
5½	51	52	1	12	55	48	124	12	0	26	76
5⅜	51	15	1	14	54	0	126	0	0	28	79
5¼	50	35	1	16	52	12	127	48	0	29	83
5⅛	49	52	1	17	50	22	129	38	0	31	86
5	49	5	1	19	48	31	131	29	0	33	90
4⅞	48	14	1	22	46	38	133	22	0	36	95
4¾	47	18	1	25	44	45	135	15	0	38	99
4⅝	46	18	1	28	42	49	137	11	0	41	105
4½	45	12	1	31	40	51	139	9	0	44	112
4⅜	44	0	1	35	38	51	141	9	0	47	119
4¼	42	41	1	39	36	48	143	12	0	50	128
4⅛	41	14	1	44	34	42	145	18	0	54	139
4	39	38	1	50	32	32	147	28	0	59	151
3⅞	37	50	1	57	30	17	149	43	1	4	165
3¾	35	48	2	6	27	56	152	4	1	10	184
3⅝	33	30	2	17	25	28	154	32	1	18	208
3½	30	51	2	30	22	49	157	11	1	28	241
3⅜	27	43	2	49	19	57	160	3	1	41	288
3¼	23	54	3	17	16	43	163	17	2	0	367
3⅛	18	56	4	6	12	53	167	7	2	31	549
3	11	23	6	6	7	31	172	29	3	50	817
*	0	0	0	0	0	0	180	0	0	0	

* The Course in the Inters. with the Equat. is equal to 33°.

The Inclin. to the Equat. is = 58°.	[redacted]					The Latitude of Vertex is = 58°.					
Course	Lat.		diff.		Long. from the Intrs.				diff.		Dist.
Points	°	′	°	′	°	′ &	°	′	°	′	Miles
8	58	0	1	0	90				0	0	53
7⅞	57	59	1	0	88	21	91	39	0	1	53
7¾	57	57	1	1	86	41	93	19	0	2	53
7⅝	57	54	1	0	85	1	94	59	0	4	53
7½	57	50	1	0	83	22	96	38	0	4	53
7⅜	57	44	1	0	81	42	98	18	0	5	54
7¼	57	36	1	1	80	2	99	58	0	7	54
7⅛	57	28	1	1	78	22	101	38	0	8	55
7	57	18	1	1	76	42	103	18	0	9	56
6⅞	57	6	1	2	75	2	104	58	0	10	56
6¾	56	53	1	3	73	21	106	39	0	11	57
6⅝	56	39	1	3	71	40	108	20	0	12	59
6½	56	22	1	4	69	59	110	1	0	13	59
6⅜	56	5	1	4	68	17	111	43	0	15	60
6¼	55	45	1	5	66	36	113	24	0	15	62
6⅛	55	23	1	7	64	53	115	7	0	17	63
6	55	0	1	7	63	11	116	49	0	18	65
5⅞	54	34	1	9	61	27	118	33	0	20	66
5¾	54	7	1	9	59	43	120	17	0	22	69
5⅝	53	37	1	10	57	59	122	1	0	23	71
5½	53	4	1	12	56	14	123	46	0	24	73
5⅜	52	29	1	13	54	28	125	32	0	26	76
5¼	51	51	1	15	52	41	127	19	0	28	79
5⅛	51	9	1	17	50	53	129	7	0	30	82
5	50	24	1	19	49	4	130	56	0	32	86
4⅞	49	36	1	21	47	14	132	46	0	34	90
4¾	48	43	1	24	45	23	134	37	0	36	94
4⅝	47	46	1	26	43	30	136	30	0	38	100
4½	46	43	1	30	41	35	138	25	0	41	106
4⅜	45	35	1	34	39	38	140	22	0	44	113
4¼	44	20	1	38	37	38	142	22	0	47	120
4⅛	42	58	1	42	35	36	144	24	0	51	130
4	41	28	1	47	33	31	146	29	0	54	141
3⅞	39	47	1	53	31	21	148	39	0	59	154
3¾	37	54	2	1	29	6	150	54	1	5	170
3⅝	35	47	2	10	26	46	153	14	1	11	190
3½	33	21	2	22	24	17	155	43	1	19	217
3⅜	30	32	2	38	21	38	158	22	1	29	253
3¼	27	11	2	59	18	43	161	17	1	43	307
3⅛	23	2	3	32	15	24	164	36	2	5	403
3	17	29	4	32	11	21	168	39	2	43	685
2⅞	7	54	7	48	4	58	175	2	4	45	560
*	0	0	0	0	0	0	180	0	0	0	

* The Course in the Inters. with the Equat. is equal to 32°.

The Inclin. to the Equat. is = 59°. The Latitude of Vertex is = 59°.

Course	Lat.		diff.		Long. from the Intrs.		&		diff.		Dist.
Points	°	'	°	'	°	'	°	'	°	'	Miles
8	59	0	1	0	90				0	0	
7⅞	58	59	1	0	88	22	91	38	0	1	51
7¾	58	58	1	0	86	43	93	17	0	2	50
7⅝	58	54	1	1	85	5	94	55	0	3	51
7½	58	50	1	0	83	26	96	34	0	4	52
7⅜	58	44	1	1	81	47	98	13	0	5	51
7¼	58	37	1	1	80	9	99	51	0	6	52
7⅛	58	29	1	1	78	30	101	30	0	7	52
7	58	19	1	2	76	51	103	9	0	8	53
6⅞	58	8	1	2	75	12	104	48	0	9	53
6¾	57	56	1	2	73	32	106	28	0	10	54
6⅝	57	42	1	3	71	52	108	8	0	12	55
6½	57	26	1	4	70	12	109	48	0	13	56
6⅜	57	9	1	5	68	32	111	28	0	14	57
6¼	56	50	1	5	66	51	113	9	0	15	58
6⅛	56	30	1	6	65	10	114	50	0	17	59
6	56	7	1	7	63	29	116	31	0	18	61
5⅞	55	43	1	8	61	47	118	13	0	19	62
5¾	55	16	1	9	60	5	119	55	0	20	64
5⅝	54	47	1	11	58	22	121	38	0	21	65
5½	54	16	1	12	56	38	123	22	0	23	68
5⅜	53	42	1	13	54	54	125	6	0	25	70
5¼	53	6	1	15	53	9	126	51	0	26	73
5⅛	52	26	1	17	51	23	128	37	0	28	75
5	51	43	1	19	49	36	130	24	0	30	78
4⅞	50	57	1	21	47	48	132	12	0	32	82
4¾	50	7	1	23	45	59	134	1	0	33	86
4⅝	49	12	1	26	44	8	135	52	0	36	90
4½	48	13	1	29	42	16	137	44	0	38	95
4⅜	47	9	1	32	40	22	139	38	0	41	100
4¼	45	58	1	36	38	25	141	35	0	44	107
4⅛	44	40	1	40	36	27	143	33	0	47	114
4	43	15	1	45	34	25	145	35	0	51	122
3⅞	41	40	1	51	32	20	147	40	0	55	132
3¾	39	55	1	58	30	11	149	49	1	0	143
3⅝	37	57	2	6	27	57	152	3	1	5	158
3½	35	43	2	16	25	36	154	24	1	12	175
3⅜	33	10	2	28	23	7	156	53	1	20	197
3¼	30	10	2	46	20	26	159	34	1	31	226
3⅛	26	34	3	10	17	29	162	31	1	46	266
3	22	1	3	50	14	4	165	56	2	10	331
2⅞	15	42	5	8	9	43	170	17	2	59	452
*	0	0	0	0	0	0	180	0	0	0	1104

* The Course in the Inters. with the Equat. is equal to 81°.

The inclin. to the Equat. is = 60°. The Latitude of Vertex is = 60°.

Course	Lat.		diff.		Long. from the Intrs.		&		diff.		Dist.
Points	°	'	°	'	°	'	°	'	°	'	Miles
8	60	0	1	0	90				0	0	
7⅞	59	59	1	0	88	23	91	37	0	1	49
7¾	59	58	1	0	86	45	93	15	0	2	48
7⅝	59	55	1	0	85	8	94	52	0	3	49
7½	59	50	1	1	83	30	96	30	0	4	50
7⅜	59	45	1	1	81	52	98	8	0	5	49
7¼	59	38	1	1	80	15	99	45	0	6	50
7⅛	59	30	1	1	78	37	101	23	0	7	50
7	59	21	1	2	76	59	103	1	0	8	51
6⅞	59	10	1	2	75	21	104	39	0	9	51
6¾	58	58	1	3	73	42	106	18	0	10	52
6⅝	58	45	1	3	72	4	107	56	0	11	53
6½	58	30	1	4	70	25	109	35	0	12	53
6⅜	58	14	1	4	68	46	111	14	0	13	55
6¼	57	55	1	5	67	6	112	54	0	15	55
6⅛	57	36	1	6	65	27	114	33	0	15	57
6	57	14	1	7	63	47	116	13	0	16	58
5⅞	56	51	1	7	62	6	117	54	0	18	60
5¾	56	25	1	9	60	25	119	35	0	19	61
5⅝	55	58	1	10	58	43	121	17	0	21	63
5½	55	28	1	11	57	1	122	59	0	22	64
5⅜	54	55	1	13	55	19	124	41	0	23	67
5¼	54	21	1	14	53	35	126	25	0	25	70
5⅛	53	43	1	16	51	51	128	9	0	26	72
5	53	2	1	18	50	6	129	54	0	28	75
4⅞	52	18	1	20	48	20	131	40	0	30	78
4¾	51	30	1	22	46	32	133	28	0	32	81
4⅝	50	38	1	25	44	44	135	16	0	34	86
4½	49	42	1	27	42	54	137	6	0	36	90
4⅜	48	41	1	30	41	3	138	57	0	38	95
4¼	47	34	1	34	39	9	140	51	0	41	101
4⅛	46	20	1	39	37	14	142	46	0	44	108
4	45	0	1	43	35	16	144	44	0	47	115
3⅞	43	31	1	48	33	15	146	45	0	51	124
3¾	41	53	1	54	31	11	148	49	0	55	134
3⅝	40	3	2	2	29	2	150	58	1	0	147
3½	37	59	2	11	26	48	153	12	1	6	162
3⅜	35	38	2	22	24	27	155	33	1	13	180
3¼	32	56	2	36	21	57	158	3	1	22	204
3⅛	29	44	2	55	19	15	160	45	1	33	237
3	25	51	3	23	16	14	163	46	1	50	283
2⅞	20	50	4	11	12	42	167	18	2	17	358
2¾	13	27	5	59	7	56	172	4	3	21	520
*	0	0	0	0	0	0	180	0	0	0	935

* The Course in the Inters. with the Equat. is equal to 80°.

The Inclin. to the Equat. is = 61°			The Latitude of Vertex is = 61°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
6¾	60 1	1 2	73 52	106 8	0 10	
6⅝	59 48	1 3	72 15	107 45	0 10	51
6½	59 34	1 3	70 37	109 23	0 11	51 52
6⅜	59 18	1 4	68 59	111 1	0 12	
6¼	59 0	1 5	67 21	112 39	0 13	53
6⅛	58 42	1 5	65 43	114 18	0 15	55
6	58 21	1 7	64 3	115 57	0 16	55 57
5⅞	57 58	1 8	62 24	117 36	0 17	
5¾	57 34	1 9	60 44	119 16	0 18	59
5⅝	57 8	1 10	59 4	120 56	0 19	60
5½	56 39	1 11	57 23	122 37	0 21	62 64
5⅜	56 8	1 13	55 42	124 18	0 22	
5¼	55 35	1 14	54 0	126 0	0 23	66
5⅛	54 59	1 16	52 17	127 43	0 25	69
5	54 20	1 17	50 34	129 26	0 26	71 75
4⅞	53 38	1 19	48 50	131 10	0 28	
4¾	52 52	1 22	47 4	132 56	0 30	77
4⅝	52 3	1 24	45 18	134 42	0 32	82
4½	51 9	1 27	43 30	136 30	0 34	86 90
4⅜	50 11	1 30	41 41	138 19	0 36	
4¼	49 8	1 33	39 50	140 10	0 39	96
4⅛	47 59	1 37	37 58	142 2	0 41	101
4	46 43	1 41	36 3	143 57	0 44	109 116
3⅞	45 19	1 46	34 6	145 54	0 47	
3¾	43 47	1 52	32 6	147 54	0 51	126
3⅝	42 5	1 58	30 2	149 58	0 55	137
3½	40 10	2 6	27 54	152 6	1 0	150 166
3⅜	38 0	2 16	25 40	154 20	1 6	
3¼	35 32	2 28	23 19	156 41	1 13	187
3⅛	32 39	2 44	20 48	159 12	1 23	213
3	29 14	3 6	18 4	161 56	1 36	248 302
2⅞	25 1	3 38	14 59	165 1	1 55	393
2¾	19 26	4 37	11 17	168 43	2 27	628
2⅝	10 24	7 20	5 50	174 10	3 57	714
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 29°.

The Inclin. to the Equat. is = 62°			The Latitude of Vertex is = 62°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
6¼	60 5	1 5	67 34	112 26	0 13	
6⅛	59 47	1 6	65 57	114 3	0 14	52
6	59 28	1 6	64 19	115 41	0 15	53 55
5⅞	59 6	1 8	62 41	117 19	0 16	
5¾	58 43	1 8	61 2	118 58	0 17	56
5⅝	58 18	1 9	59 23	120 37	0 19	58
5½	57 50	1 11	57 44	122 16	0 19	59 61
5⅜	57 21	1 12	56 4	123 56	0 21	
5¼	56 49	1 14	54 23	125 37	0 23	63
5⅛	56 15	1 15	52 42	127 18	0 24	66
5	55 37	1 17	51 0	129 0	0 25	68 71
4⅞	54 57	1 19	49 18	130 42	0 27	
4¾	54 14	1 21	47 34	132 26	0 29	74
4⅝	53 27	1 23	45 50	134 10	0 30	77
4½	52 36	1 26	44 4	135 56	0 32	81 86
4⅜	51 41	1 29	42 17	137 43	0 34	
4¼	50 41	1 32	40 29	139 31	0 36	91
4⅛	49 36	1 35	38 39	141 21	0 39	96
4	48 24	1 39	36 47	143 13	0 41	102 110
3⅞	47 5	1 44	34 53	145 7	0 45	
3¾	45 39	1 49	32 57	147 3	0 47	118
3⅝	44 3	1 55	30 57	149 3	0 51	128
3½	42 16	2 2	28 54	151 6	0 55	140 154
3⅜	40 16	2 11	26 46	153 14	1 1	
3¼	38 0	2 21	24 32	155 28	1 7	171
3⅛	35 23	2 35	22 11	157 49	1 14	193
3	32 20	2 52	19 40	160 20	1 24	223 262
2⅞	28 39	3 18	16 54	163 6	1 37	
2¾	24 3	3 56	13 44	166 16	1 59	325
2⅝	17 44	5 11	9 47	170 13	2 39	438
2½	5 11	10 26	2 46	177 14	5 25	859
*	0 0	0 0	0 0	180 0	0 0	352

* The Course in the Inters. with the Equat. is equal to 28°.

(X.)

EXAMPLES ON THE DETERMINATION OF THE GREAT CIRCLE BY CALCULATION.

Ex. 1. To determine by calculation the Great Circle which passes through St. Helena, Lat. 15° 55′ S., Long. 5° 44′ W., and C. Horn, Lat. 55° 59′ S., Long. 67° 16′ W.

1. To find the longitudes from the Inters. with the Equator.

The D. Long. is 61° 32′; half D. 30° 46′ log. tan. 9·7748
(C. Horn) lat. 55° 59′ S.
(St. Helena) lat. 15 55 S.
sum 71 54 log. sin. 9·9780
diff. 40 4 log. cosec. 10·1913
(required longits.) half Sum 41° 19′ log. tan. 9·9441
** C.H. Long. fr. the Int. (sum) 72 5 W.
* St.H. Long. fr. the Int. (diff.) 10 33 W.

2. To find the Incl. to the Eq. (Lat. of Vertex.)

Employing St. Helena.
* Table's long. 10° 33′ log. sin. 9·2627
lat. 15° 55′ log. cot. 10·5449
required Incl. 57° 18′ log. cot. 9·8076

Employing C. Horn.
** Table's long. 72° 5′ log. sin. 9·9784
lat. 55° 59′ log. cot. 9·8293
required Incl. 57° 18′ log. cot. 9·8077

The Inclin. to the Equat. is = 63°.			The Latitude of Vertex is = 63°.			
Course	**Lat.**	**diff.**	**Long. from the Intrs.**		**diff.**	**Dist.**
Points	° ′	° ′	° ′	° ′	° ′	Miles
5 7/8	60 14	1 7	62 57	117 3	0 15	54
5 3/4	59 51	1 9	61 19	118 41	0 17	55
5 5/8	59 27	1 10	59 42	120 18	0 17	
5 1/2	59 1	1 11	58 3	121 57	0 19	56 59
5 3/8	58 33	1 12	56 25	123 35	0 20	60
5 1/4	58 3	1 13	54 46	125 14	0 21	62
5 1/8	57 30	1 15	53 6	126 54	0 22	
5	56 54	1 17	51 25	128 35	0 24	65 68
4 7/8	56 16	1 19	49 45	130 15	0 25	70
4 3/4	55 35	1 20	48 3	131 57	0 26	74
4 5/8	54 50	1 23	46 20	133 40	0 28	
4 1/2	54 2	1 25	44 36	135 24	0 30	77 81
4 3/8	53 10	1 27	42 51	137 9	0 32	86
4 1/4	52 13	1 30	41 5	138 55	0 34	91
4 1/8	51 11	1 34	39 18	140 42	0 36	
4	50 3	1 38	37 28	142 32	0 39	97 103
3 7/8	48 49	1 42	35 38	144 22	0 41	111
3 3/4	47 28	1 47	33 44	146 16	0 45	120
3 5/8	45 58	1 53	31 48	148 12	0 48	
3 1/2	44 18	2 0	29 49	150 11	0 52	131 143
3 3/8	42 27	2 7	27 47	152 13	0 55	158
3 1/4	40 21	2 16	25 39	154 21	1 1	177
3 1/8	37 58	2 27	23 25	156 35	1 8	
3	35 12	2 42	21 4	158 56	1 15	201 232
2 7/8	31 57	3 2	18 31	161 29	1 26	279
2 3/4	27 59	3 31	15 43	164 17	1 40	352
2 5/8	22 55	4 17	12 26	167 34	2 5	
2 1/2	15 37	5 57	8 11	171 49	2 56	500
*	0 0	0 0	0 0	180 0	0 0	1055

* The Course in the Inters. with the Equat. is equal to 27°.

The Inclin. to the Equat. is = 64°.			The Latitude of Vertex is = 64°.			
Course	**Lat.**	**diff.**	**Long. from the Intrs.**		**diff.**	**Dist.**
Points	° ′	° ′	° ′	° ′	° ′	Miles
5 1/2	60 12	1 10	58 22	121 38	0 17	54 55
5 3/8	59 45	1 11	56 45	123 15	0 18	58
5 1/4	59 16	1 13	55 7	124 53	0 19	59
5 1/8	58 45	1 14	53 28	126 32	0 21	
5	58 11	1 16	51 49	128 11	0 23	62 64
4 7/8	57 35	1 18	50 10	129 50	0 23	67
4 3/4	56 55	1 20	48 29	131 31	0 25	70
4 5/8	56 13	1 22	46 48	133 12	0 27	
4 1/2	55 27	1 24	45 6	134 54	0 28	74 77
4 3/8	54 37	1 27	43 23	136 37	0 30	81
4 1/4	53 43	1 30	41 39	138 21	0 32	86
4 1/8	52 45	1 33	39 54	140 6	0 34	
4	51 41	1 37	38 7	141 53	0 36	91 98
3 7/8	50 31	1 41	36 19	143 41	0 38	104
3 3/4	49 15	1 45	34 29	145 31	0 41	113
3 5/8	47 51	1 50	32 36	147 24	0 44	
3 1/2	46 18	1 56	30 41	149 19	0 47	122 133
3 3/8	44 34	2 3	28 42	151 18	0 52	146
3 1/4	42 37	2 11	26 40	153 20	0 56	163
3 1/8	40 25	2 22	24 33	155 27	1 1	
3	37 54	2 34	22 19	157 41	1 8	183 209
2 7/8	34 59	2 50	19 57	160 3	1 16	246
2 3/4	31 30	3 13	17 23	162 37	1 28	298
2 5/8	27 12	3 46	14 31	165 29	1 44	
2 1/2	21 34	4 44	11 7	168 53	2 12	385 590
2 3/8	12 50	7 7	6 23	173 37	3 22	859
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 26°.

(XI.)
EXAMPLES ON THE DETERMINATION OF THE GREAT CIRCLE BY CALCULATION.

3. Determination of the geographical Longitude of the Intersection of the Great Circle with the Equator,

by means of St. Helena's { geogr. Long. 5°44′ W.
L. fr. the Inters. 10 33 W.

Geogr. L. of the Int. = (algebr. diff.) 4 49 E.

by means of C. Horn's { geogr. Long. 67°16′ W.
** L. fr. the Int. 72 5 W.

Geogr. L. of the Inters. = (algbr. diff.) 4 49 E.

Ex. 2. Being in the entrance of the English Channel, Lat. 49° 6′ N., Long. 6° 2′ W.; and bound to New-York in Lat. 40° 42′ N., Long. 74° 2′ W., to determine the Great Circle which passes through those two places.

1. To find the longitudes from the Inters. with the Equator.

The D. Long. is 68°; half D. 34° 0′ log. tan. 9·8290

(Eng. Channel) lat. 49° 6′ N.
(New-York) lat. 40 42 N.

sum 89 48 log. sin. 10·0000
diff. 8 24 log. cosec. 10·8354

(required longs.) half Sum 77°47′ log. tan. 10·6644

** E.Ch. Lg. fr. the Int. (sum) 111 47 E. = 68°13′ W.
* N.Y. Lg. fr. the Int. (diff.) 43 47 E. = 136 13 W.

2. To find the Incl. to the Eq. (Lat. of Vertex.)

Employing New-York.

* Table's long. 43°47′ log. sin. 9·8400
lat. 40°42′ log. cot. 10·0654

required Incl. 51° 12′ log. cot. 9·9054

Employing the entr. of the E. Ch.

** Table's long. 68°13′ log. sin. 9·9678
lat. 49° 6′ log. cot. 9·9376

required Incl. 51° 12′ log. cot. 9·9054

The Inclin. to the Equat. is = 65°	[redacted]	The Latitude of Vertex is = 65°			
Course	Lat.	diff.	Long. from the Intrs.	diff.	Dist.
Points	° ′	° ′	° ′ & ° ′	° ′	Miles
5¼	60 29	1 13	55 26 & 124 34	0 19	57
5⅛	59 59	1 14	53 49 & 126 11	0 20	58
5	59 27	1 16	52 12 & 127 48	0 21	61
4⅞	58 53	1 17	50 33 & 129 27	0 23	64
4¾	58 15	1 19	48 54 & 131 6	0 24	66
4⅝	57 35	1 21	47 15 & 132 45	0 25	70
4½	56 51	1 24	45 34 & 134 26	0 27	73
4⅜	56 4	1 27	43 53 & 136 7	0 28	77
4¼	55 13	1 29	42 11 & 137 49	0 30	82
4⅛	54 18	1 32	40 28 & 139 32	0 32	86
4	53 18	1 35	38 43 & 141 17	0 34	92
3⅞	52 12	1 39	36 57 & 143 3	0 36	98
3¾	51 0	1 43	35 10 & 144 50	0 38	105
3⅝	49 41	1 48	33 20 & 146 40	0 41	114
3½	48 14	1 53	31 28 & 148 32	0 44	124
3⅜	46 37	2 0	29 34 & 150 26	0 47	136
3¼	44 48	2 8	27 36 & 152 24	0 51	150
3⅛	42 47	2 17	25 34 & 154 26	0 56	168
3	40 28	2 28	23 27 & 156 33	1 1	190
2⅞	37 49	2 42	21 13 & 158 47	1 9	220
2¾	34 43	2 59	18 51 & 161 9	1 17	259
2⅝	30 58	3 26	16 15 & 163 45	1 30	320
2½	26 18	4 4	13 19 & 166 41	1 48	429
2⅜	19 57	5 17	9 45 & 170 15	2 21	750
2¼	8 43	9 14	4 6 & 175 54	4 12	577
*	0 0	0 0	0 0 & 180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 25°.

The Inclin. to the Equat. is = 66°	[redacted]	The Latitude of Vertex is = 66°			
Course	Lat.	diff.	Long. from the Intrs.	diff.	Dist.
Points	° ′	° ′	° ′ & ° ′	° ′	Miles
5	60 43	1 15	52 33 & 127 27	0 20	58
4⅞	60 10	1 17	50 56 & 129 4	0 21	61
4¾	59 34	1 19	49 18 & 130 42	0 22	63
4⅝	58 56	1 21	47 40 & 132 20	0 24	66
4½	58 15	1 23	46 1 & 133 59	0 25	69
4⅜	57 31	1 25	44 21 & 135 39	0 27	73
4¼	56 42	1 28	42 41 & 137 19	0 28	77
4⅛	55 50	1 31	41 0 & 139 0	0 29	81
4	54 53	1 34	39 17 & 140 43	0 32	87
3⅞	53 51	1 38	37 33 & 142 27	0 34	92
3¾	52 43	1 42	35 48 & 144 12	0 36	99
3⅝	51 29	1 47	34 1 & 145 59	0 38	107
3½	50 7	1 52	32 12 & 147 48	0 41	116
3⅜	48 37	1 57	30 21 & 149 39	0 44	127
3¼	46 56	2 4	28 27 & 151 33	0 47	139
3⅛	45 4	2 12	26 30 & 153 30	0 51	154
3	42 56	2 22	24 28 & 155 32	0 56	173
2⅞	40 31	2 34	22 22 & 157 38	1 2	198
2¾	37 42	2 50	20 8 & 159 52	1 9	230
2⅝	34 24	3 10	17 45 & 162 15	1 18	276
2½	30 22	3 39	15 7 & 164 53	1 32	348
2⅜	25 14	4 25	12 6 & 167 54	1 53	485
2¼	17 57	6 0	8 18 & 171 42	2 34	1183
*	0 0	0 0	0 0 & 180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 24°.

(XII.)

EXAMPLES ON THE DETERMINATION OF THE GREAT CIRCLE BY CALCULATION.

3. Determination of the geographical Longitude of the intersection of the Great Circle with the Equator,

by means of the adopted place's (Engl. Channel) { geogr. Long. 6° 2′ W.
**L.fr.the Int. 68 13 W.
Ggr. Long. of the Int. = (algebr. diff.) 62 11 E.

by means of N. York's { geogr. Long. 74° 2′ W.
* L. fr. the Int. 136 13 W.
Ggr. Lg. of the Int. = (algebr.diff.) 62 11 E.

Ex. 3. To determine by calculation the Great Circle which passes through Panama, Lat. 8° 57′ N., Long. 79° 31′ W., and Port Jackson, Lat. 33° 51′ S., Long. 151° 18′ E.

1. To find the longs. from the Inters. with the Equator.

The D.L. is 230° 49′, half D. 115° 24½′		log. tan.	10·3233
(Port Jackson) lat.	33° 51′ S.		
(Panama) lat.	8 57 N.		
sum	24 54	log. sin.	9·6243
diff.	42 48	log. cosec.	10·1678
(requir. longs.) half Sum	52° 31½′	log. tan.	10·1154

* Pa. L. fr. the Int. (sum) 167 56 W. = 12° 4′ E.
** P.J. L. fr. the Int. (diff.) 62 53 E. = 117 7 W.

2. To find the Incl. to the Eq. (Lat. of Vertex.)

Employing Panama.

* Table's long. 12° 4′	log. sin.	9·3202
lat. 8° 57′	log. cot.	10·8027
required Incl. 37° 0′	log. cot.	10·1229

Employing Port Jackson.

**Table's long. 62° 53′	log. sin.	9·9494
lat. 33° 51′	log. cot.	10·1735
required Incl. 37° 0′	log. cot.	10·1229

3. Determination of the geographical Longitude of the Intersection of the Great Circle with the Equator,

by means of Panama's { geogr. Long. 79° 31′ W.
* L. fr. the Inters. 12 4 E.
Geogr. L. of the Int. = (algebr. diff.) 91 35 W.

by means of P. Jackson's { geogr. Long. 208° 52′ W.
**L.fr.the Int. 117 7 W.
Geogr. L. of the Int. = (algebr. diff.) 91 35 W.

The Inclin. to the Equat. is = 67°.			The Latitude of Vertex is = 67°.			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
$4^5/_8$	60 17	1 21	48 4	131 56	0 22	62
$4^1/_2$	59 38	1 23	46 26	133 34	0 24	66
$4^3/_8$	58 56	1 25	44 48	135 12	0 25	69
$4^1/_4$	58 10	1 28	43 9	136 51	0 26	72
$4^1/_8$	57 21	1 30	41 29	138 31	0 28	77
4	56 27	1 34	39 49	140 11	0 29	82
$3^7/_8$	55 29	1 37	38 7	141 53	0 31	87
$3^3/_4$	54 25	1 41	36 24	143 36	0 33	93
$3^5/_8$	53 16	1 44	34 39	145 21	0 36	100
$3^1/_2$	51 59	1 49	32 53	147 7	0 38	108
$3^3/_8$	50 34	1 55	31 5	148 55	0 41	118
$3^1/_4$	49 0	2 2	29 14	150 46	0 44	129
$3^1/_8$	47 16	2 9	27 21	152 39	0 47	142
3	45 18	2 18	25 24	154 36	0 52	159
$2^7/_8$	43 5	2 28	23 24	156 36	0 56	180
$2^3/_4$	40 32	2 42	21 17	158 43	1 2	207
$2^5/_8$	37 34	2 58	19 3	160 57	1 10	242
$2^1/_2$	34 1	3 22	16 39	163 21	1 20	295
$2^3/_8$	29 39	3 55	13 59	166 1	1 34	381
$2^1/_4$	23 57	4 52	10 52	169 8	1 59	565
$2^1/_8$	15 23	7 2	6 42	173 18	2 54	1005
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 23°.

The Inclin. to the Equat. is = 68°.			The Latitude of Vertex is = 68°.			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
$4^3/_8$	60 21	1 24	45 13	134 47	0 23	65
$4^1/_4$	59 38	1 27	43 35	136 25	0 25	68
$4^1/_8$	58 51	1 30	41 57	138 3	0 26	73
4	58 1	1 32	40 18	139 42	0 28	77
$3^7/_8$	57 6	1 36	38 38	141 22	0 29	81
$3^3/_4$	56 6	1 39	36 57	143 3	0 31	88
$3^5/_8$	55 0	1 43	35 15	144 45	0 33	94
$3^1/_2$	53 48	1 48	33 31	146 29	0 35	101
$3^3/_8$	52 29	1 53	31 46	148 14	0 37	109
$3^1/_4$	51 2	1 59	29 58	150 2	0 41	120
$3^1/_8$	49 25	2 6	28 8	151 52	0 44	132
3	47 36	2 14	26 16	153 44	0 47	147
$2^7/_8$	45 33	2 24	24 20	155 40	0 51	164
$2^3/_4$	43 14	2 35	22 19	157 41	0 56	187
$2^5/_8$	40 32	2 50	20 13	159 47	1 2	216
$2^1/_2$	37 23	3 8	17 59	162 1	1 10	257
$2^3/_8$	33 34	3 35	15 33	164 27	1 21	318
$2^1/_4$	28 49	4 14	12 51	167 9	1 37	422
$2^1/_8$	22 25	5 25	9 36	170 24	2 6	693
2	11 48	8 44	4 50	175 10	3 26	764
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 22°.

(XIII.)
SKETCHES TO ILLUSTRATE THE PRECEDING EXAMPLES.

The accompanying three sketches are made to shew the position of the points of intersection of the Equator with the great circle passing through the places adopted in every one of the preceding examples. That position is referred to the meridian of Greenwich.

The instruction to make such a sketch is given Art. 50 and is there specially applied in making the three given sketches, of which,

Fig. 22 contains the great circle which passes through St. Helena and Cap Horn (Ex. 1);

Fig. 23 the great circle which passes through the adopted ship's place in the entrance of the English Channel and New York (Ex. 2);
and
Fig. 24 that which passes through Panama and Port Jackson (Ex. 3).

Fig. 22.

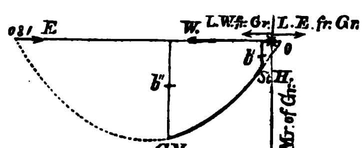


Fig. 23.

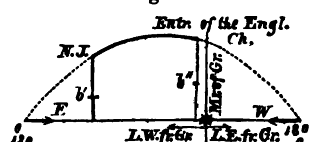
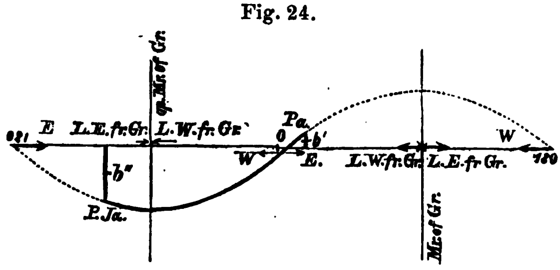


Fig. 24.



The Inclin. to the Equat. is = 69°. The Latitude of Vertex is = 69°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
4 1/8	60 21	1 28	42 23	137 37	0 25	
4	59 33	1 31	40 46	139 14	0 26	68
3 7/8	58 41	1 35	39 7	140 53	0 28	73
3 3/4	57 45	1 38	37 28	142 32	0 29	76
3 5/8	56 43	1 42	35 48	144 12	0 31	82
3 1/2	55 36	1 46	34 6	145 54	0 33	88
3 3/8	54 22	1 52	32 23	147 37	0 35	95
3 1/4	53 1	1 57	30 39	149 21	0 37	102
3 1/8	51 31	2 3	28 52	151 8	0 40	112
3	49 50	2 10	27 3	152 57	0 43	122
2 7/8	47 57	2 19	25 11	154 49	0 47	135
2 3/4	45 49	2 29	23 15	156 45	0 51	150
2 5/8	43 22	2 42	21 15	158 45	0 56	170
2 1/2	40 31	2 58	19 9	160 51	1 3	195
2 3/8	37 9	3 19	16 54	163 6	1 12	228
2 1/4	33 3	3 49	14 28	165 32	1 23	273
2 1/8	27 50	4 36	11 42	168 18	1 40	245
2	20 32	6 7	8 16	171 44	2 16	476
1 7/8	5 17	12 51	2 2	177 58	4 49	985
*	0 0	0 0	0 0	180 0	0 0	339

* The Course in the Inters. with the Equat. is equal to 21°.

The Inclin. to the Equat. is = 70°. The Latitude of Vertex is = 70°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
4	61 4	1 31	41 12	138 48	0 24	
3 7/8	60 16	1 34	39 35	140 25	0 25	68
3 3/4	59 23	1 37	37 57	142 3	0 27	72
3 5/8	58 25	1 41	36 19	143 41	0 28	77
3 1/2	57 22	1 44	34 39	145 21	0 31	82
3 3/8	56 14	1 49	32 58	147 2	0 33	88
3 1/4	55 58	1 54	31 16	148 44	0 35	95
3 1/8	53 34	2 0	29 32	150 28	0 37	104
3	52 0	2 7	27 46	152 14	0 40	113
2 7/8	50 16	2 15	25 58	154 2	0 43	125
2 3/4	48 18	2 24	24 6	155 54	0 47	138
2 5/8	46 4	2 36	22 11	157 49	0 51	156
2 1/2	43 29	2 50	20 12	159 48	0 56	176
2 3/8	40 28	3 8	18 6	161 54	1 3	204
2 1/4	36 52	3 32	15 51	164 9	1 12	240
2 1/8	32 26	4 7	13 22	166 38	1 25	293
2	26 39	5 3	10 32	169 28	1 45	377
1 7/8	18 8	7 6	6 51	173 9	2 29	550
*	0 0	0 0	0 0	180 0	0 0	1161

* The Course in the Inters. with the Equat. is equal to 20°.

The Inclin. to the Equat. is = 71° The Latitude of Vertex is = 71°

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
3 7/8	61 50	1 33	40 0	140 0	0 24	
3 3/4	61 0	1 36	38 24	141 36	0 25	68
3 5/8	60 6	1 40	36 47	143 13	0 27	72
3 1/2	59 7	1 44	35 10	144 50	0 28	76
3 3/8	58 3	1 48	33 31	146 29	0 30	83
3 1/4	56 52	1 53	31 51	148 9	0 32	89
3 1/8	55 34	1 59	30 9	149 51	0 34	96
3	54 7	2 5	28 26	151 34	0 37	105
2 7/8	52 31	2 12	26 41	153 19	0 39	115
2 3/4	50 42	2 21	24 53	155 7	0 43	128
2 5/8	48 40	2 31	23 2	156 58	0 47	142
2 1/2	46 19	2 43	21 8	158 52	0 51	160
2 3/8	43 36	2 59	19 9	160 51	0 56	184
2 1/4	40 24	3 19	17 3	162 57	1 3	213
2 1/8	36 33	3 46	14 47	165 13	1 13	255
2	31 42	4 27	12 17	167 43	1 27	316
1 7/8	25 14	5 36	9 20	170 40	1 51	418
1 3/4	14 54	8 34	5 15	174 45	2 52	661
*	0 0	0 0	0 0	180 0	0 0	947

* The Course in the Inters. with the Equat. is equal to 19°.

The Inclin. to the Equat. is = 72° The Latitude of Vertex is = 72°

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
3 3/4	62 36	1 36	38 49	141 11	0 24	
3 5/8	61 46	1 39	37 14	142 46	0 25	68
3 1/2	60 51	1 43	35 38	144 22	0 26	71
3 3/8	59 51	1 47	34 1	145 59	0 28	77
3 1/4	58 45	1 51	32 23	147 37	0 29	83
3 1/8	57 33	1 56	31 43	148 17	0 32	89
3	56 12	2 3	29 3	150 57	0 33	98
2 7/8	54 43	2 9	27 20	152 40	0 36	106
2 3/4	53 3	2 17	25 36	154 24	0 39	118
2 5/8	51 11	2 26	23 49	156 11	0 42	130
2 1/2	49 2	2 38	21 59	158 1	0 46	146
2 3/8	46 35	2 51	20 5	159 55	0 50	166
2 1/4	43 43	3 8	18 6	161 54	0 56	191
2 1/8	40 19	3 30	16 0	164 0	1 4	225
2	36 9	4 2	13 44	166 16	1 14	272
1 7/8	30 50	4 50	11 11	168 49	1 30	343
1 3/4	23 28	6 19	8 7	171 53	1 58	472
1 5/8	9 54	11 20	3 15	176 45	3 35	861
*	0 0	0 0	0 0	180 0	0 0	624

* The Course in the Inters. with the Equat. is equal to 18°.

The Inclin. to the Equat. is = 73° | The Latitude of Vertex is = 73°

Course	Lat.		diff.		Long. from the Intrs.		&		diff.		Dist.
Points	°	′	°	′	°	′	°	′	°	′	Miles
3 5/8	63	25	1	37	37	39	142	21	0	23	
3 1/2	62	34	1	41	36	4	143	56	0	24	67
3 3/8	61	38	1	45	34	29	145	31	0	25	72
3 1/4	60	36	1	50	32	52	147	8	0	27	77
3 1/8	59	29	1	55	31	15	148	45	0	29	83
3	58	15	2	0	29	36	150	24	0	31	90
2 7/8	56	52	2	7	27	56	152	4	0	33	98
2 3/4	55	20	2	15	26	15	153	45	0	35	108
2 5/8	53	37	2	23	24	31	155	29	0	38	120
2 1/2	51	40	2	33	22	45	157	15	0	42	133
2 3/8	49	26	2	45	20	55	159	5	0	46	151
2 1/4	46	51	3	1	19	2	160	58	0	51	172
2 1/8	43	49	3	20	17	4	162	56	0	56	200
2	40	11	3	44	14	58	165	2	1	4	238
1 7/8	35	40	4	21	12	41	167	19	1	15	293
1 3/4	29	47	5	19	10	5	169	55	1	33	375
1 5/8	21	14	7	17	6	50	173	10	2	8	542
*	0	0	0	0	0	0	180	0	0	0	1336

* The Course in the Inters. with the Equat. is equal to 17°.

The Inclin. to the Equat. is = 74° | The Latitude of Vertex is = 74°

Course	Lat.		diff.		Long. from the Intrs.		&		diff.		Dist.
Points	°	′	°	′	°	′	°	′	°	′	Miles
3 1/2	64	15	1	40	36	28	143	32	0	23	
3 3/8	63	23	1	44	34	54	145	6	0	24	66
3 1/4	62	26	1	49	33	19	146	41	0	26	72
3 1/8	61	24	1	53	31	44	148	16	0	27	77
3	60	15	1	59	30	7	149	53	0	29	83
2 7/8	58	59	2	5	28	29	151	31	0	31	90
2 3/4	57	35	2	11	26	50	153	10	0	33	100
2 5/8	56	0	2	19	25	9	154	51	0	35	110
2 1/2	54	13	2	29	23	27	156	33	0	37	122
2 3/8	52	11	2	40	21	41	158	19	0	41	137
2 1/4	49	52	2	53	19	53	160	7	0	45	155
2 1/8	47	9	3	9	18	0	162	0	0	50	179
2	43	55	3	31	16	2	163	58	0	56	211
1 7/8	40	1	4	0	13	56	166	4	1	4	252
1 3/4	35	6	4	42	11	38	168	22	1	16	315
1 5/8	28	31	5	53	8	58	171	2	1	36	418
1 1/2	18	17	8	38	5	26	174	34	2	23	643
*	0	0	0	0	0	0	180	0	0	0	1143

* The Course in the Inters. with the Equat. is equal to 16°.

The Inclin. to the Equat. is = 75° | The Latitude of Vertex is = 75°

Course	Lat.		diff.		Long. from the Intrs.		&		diff.		Dist.
Points	°	′	°	′	°	′	°	′	°	′	Miles
3 3/8	65	7	1	44	35	18	144	42	0	22	
3 1/4	64	15	1	47	33	45	146	15	0	23	66
3 1/8	63	17	1	52	32	11	147	49	0	24	71
3	62	14	1	57	30	36	149	24	0	26	76
2 7/8	61	4	2	3	29	0	151	0	0	28	84
2 3/4	59	46	2	10	27	23	152	37	0	29	91
2 5/8	58	19	2	17	25	44	154	16	0	32	101
2 1/2	56	42	2	25	24	4	155	56	0	35	111
2 3/8	54	51	2	36	22	22	157	38	0	37	125
2 1/4	52	45	2	47	20	38	159	22	0	40	141
2 1/8	50	18	3	3	18	50	161	10	0	45	161
2	47	26	3	21	16	58	163	2	0	50	187
1 7/8	44	1	3	45	15	0	165	0	0	56	222
1 3/4	39	48	4	18	12	54	167	6	1	5	270
1 5/8	34	24	5	8	10	34	169	26	1	19	342
1 1/2	26	55	6	38	7	49	172	11	1	42	471
1 3/8	13	58	10	56	3	49	176	11	2	50	808
*	0	0	0	0	0	0	180	0	0	0	869

* The Course in the Inters. with the Equat. is equal to 87°.

The Inclin. to the Equat is = 76° | The Latitude of Vertex is = 76°

Course	Lat.		diff.		Long. from the Intrs.		&		diff.		Dist.
Points	°	′	°	′	°	′	°	′	°	′	Miles
3 1/8	65	9	1	52	32	35	147	25	0	22	
3	64	11	1	56	31	2	148	58	0	23	71
2 7/8	63	7	2	1	29	28	150	32	0	25	76
2 3/4	61	56	2	7	27	52	152	8	0	27	84
2 5/8	60	36	2	15	26	16	153	44	0	29	92
2 1/2	59	7	2	23	24	39	155	21	0	31	101
2 3/8	57	27	2	32	22	59	157	1	0	34	113
2 1/4	55	32	2	43	21	18	158	42	0	37	128
2 1/8	53	21	2	56	19	35	160	25	0	39	145
2	50	47	3	13	17	48	162	12	0	44	166
1 7/8	47	46	3	33	15	56	164	4	0	49	196
1 3/4	44	6	4	0	13	59	166	1	0	56	234
1 5/8	39	32	4	39	11	53	168	7	1	5	290
1 1/2	33	33	5	39	9	31	170	29	1	20	377
1 3/8	24	54	7	36	6	39	173	21	1	48	540
1 1/4	5	21	16	52	1	20	178	40	4	5	1212
*	0	0	0	0	0	0	180	0	0	0	331

* The Course in the Inters. with the Equat. is equal to 14°.

The Inclin. to the Equat. is = 77°			The Latitude of Vertex is = 77°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
3	66 7	1 54	31 25	148 35	0 22	
$2^7/_8$	65 8	2 0	29 53	150 7	0 23	70
$2^3/_4$	64 3	2 6	28 19	151 41	0 25	76
$2^5/_8$	62 51	2 12	26 45	153 15	0 26	84
$2^1/_2$	61 30	2 20	25 10	154 50	0 28	92
$2^3/_8$	59 59	2 28	23 33	156 27	0 30	103
$2^1/_4$	58 15	2 39	21 55	158 5	0 32	115
$2^1/_8$	56 17	2 51	20 14	159 46	0 36	130
2	54 0	3 5	18 32	161 28	0 39	149
$1^7/_8$	51 19	3 24	16 45	163 15	0 44	174
$1^3/_4$	48 6	3 48	14 55	165 5	0 49	205
$1^5/_8$	44 11	4 18	12 58	167 2	0 56	249
$1^1/_2$	39 12	5 3	10 51	169 9	1 6	314
$1^3/_8$	32 30	6 17	8 27	171 33	1 23	418
$1^1/_4$	22 13	8 57	5 25	174 35	1 58	638
*	0 0	0 0	0 0	180 0	0 0	1370

* The Course in the Inters. with the Equat. is equal to 13°.

The Inclin. to the Equat. is = 78°			The Latitude of Vertex is = 78°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
$2^3/_4$	66 9	2 4	28 44	151 16	0 22	
$2^5/_8$	65 3	2 10	27 11	152 49	0 24	76
$2^1/_2$	63 50	2 17	25 38	154 22	0 25	84
$2^3/_8$	62 27	2 26	24 3	155 57	0 27	93
$2^1/_4$	60 54	2 36	22 27	157 33	0 29	103
$2^1/_8$	59 8	2 47	20 50	159 10	0 32	117
2	57 5	3 0	19 11	160 49	0 34	133
$1^7/_8$	54 43	3 16	17 29	162 31	0 38	154
$1^3/_4$	51 54	3 36	15 44	164 16	0 42	181
$1^5/_8$	48 29	4 3	13 54	166 6	0 48	216
$1^1/_2$	44 15	4 39	11 57	168 3	0 56	266
$1^3/_8$	38 47	5 32	9 50	170 10	1 7	342
$1^1/_4$	31 10	7 5	7 23	172 37	1 26	472
$1^1/_8$	18 23	11 3	4 3	175 57	2 15	788
*	0 0	0 0	0 0	180 0	0 0	1129

* The Course in the Inters. with the Equat. is equal to 12°.

The Inclin. to the Equat. is = 79°			The Latitude of Vertex is = 79°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
$2^1/_2$	66 7	2 16	26 3	153 57	0 22	
$2^3/_8$	64 53	2 24	24 30	155 30	0 24	83
$2^1/_4$	63 30	2 32	22 56	157 4	0 26	93
$2^1/_8$	61 55	2 43	21 22	158 38	0 28	104
2	60 5	2 56	19 45	160 15	0 31	119
$1^7/_8$	57 59	3 10	18 7	161 53	0 33	136
$1^3/_4$	55 30	3 28	16 26	163 34	0 37	159
$1^5/_8$	52 32	3 51	14 42	165 18	0 41	189
$1^1/_2$	48 54	4 21	13 53	166 7	0 47	228
$1^3/_8$	44 19	5 4	10 57	169 3	0 55	286
$1^1/_4$	38 15	6 8	8 49	171 11	1 7	377
$1^1/_8$	29 26	8 9	6 18	173 42	1 30	543
1	12 2	15 5	2 22	177 38	2 49	1068
*	0 0	0 0	0 0	180 0	0 0	735

* The Course in the Inters. with the Equat. is equal to 11°.

The Inclin. to the Equat. is = 80°			The Latitude of Vertex is = 80°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
$2^1/_4$	66 2	2 30	23 22	156 38	0 23	
$2^1/_8$	64 38	2 40	21 50	158 10	0 24	93
2	63 1	2 51	20 16	159 44	0 26	106
$1^7/_8$	61 9	3 5	18 40	161 21	0 29	120
$1^3/_4$	58 58	3 22	17 3	162 57	0 32	139
$1^5/_8$	56 23	3 42	15 23	164 37	0 36	165
$1^1/_2$	53 15	4 8	13 40	166 20	0 40	196
$1^3/_8$	49 23	4 42	11 52	168 8	0 46	243
$1^1/_4$	44 23	5 32	9 56	170 4	0 [illegible]5	310
$1^1/_8$	37 35	6 51	7 48	172 12	1 8	419
1	27 7	9 35	5 11	174 49	1 36	642
*	0 0	0 0	0 0	180 0	0 0	1654

* The Course in the Inters. with the Equat. is equal to 10°.

The Inclin. to the Equat. is = 81°			The Latitude of Vertex is = 81°			
Course	Lat.	diff.	Long. from the Intrs.		diff.	Dist.
Points	° ′	° ′	° ′ &	° ′	° ′	Miles
$2^1/_8$	67 18	2 37	22 14	157 46	0 22	
2	65 52	2 48	20 42	159 18	0 24	93
$1^7/_8$	64 14	3 1	19 9	160 51	0 26	105
$1^3/_4$	62 20	3 16	17 35	162 25	0 28	122
$1^5/_8$	60 5	3 35	15 59	164 1	0 31	143
$1^1/_2$	57 23	3 58	14 20	165 40	0 35	169
$1^3/_8$	54 5	4 28	12 38	167 22	0 39	207
$1^1/_4$	49 55	5 8	10 51	169 9	0 45	258
$1^1/_8$	44 26	6 8	8 56	171 4	0 54	338
1	36 42	7 47	6 47	173 13	1 9	475
$^7/_8$	23 47	11 43	4 0	176 0	1 45	788
*	0 0	0 0	0 0	180 0	0 0	1446

* The Course in the Inters. with the Equat. is equal to 9°.

The Inclin. to the Equat. is = 82°. [illegible] The Latitude of Vertex is = 82°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
2	68 40	2 46	21 6	158 54	0 20	92
1 7/8	67 15	2 57	19 35	160 25	0 22	106
1 3/4	65 36	3 11	18 3	161 57	0 24	123
1 5/8	63 40	3 28	16 30	163 30	0 26	145
1 1/2	61 21	3 50	14 55	165 5	0 29	175
1 3/8	58 33	4 16	13 17	166 43	0 33	217
1 1/4	55 3	4 51	11 36	168 24	0 38	277
1 1/8	50 34	5 38	9 50	170 10	0 44	372
1	44 29	6 51	7 56	172 4	0 54	548
7/8	35 30	9 2	5 45	174 15	1 11	1036
3/4	18 28	15 23	2 41	177 19	2 2	1119
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 8°.

The Inclin. to the Equat. is = 83°. [illegible] The Latitude of Vertex is = 83°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
1 3/4	68 47	3 8	18 27	161 33	0 20	105
1 5/8	67 8	3 24	16 56	163 4	0 22	124
1 1/2	65 11	3 43	15 24	164 36	0 24	147
1 3/8	62 49	4 7	13 50	166 10	0 27	181
1 1/4	59 54	4 37	12 14	167 46	0 31	228
1 1/8	56 12	5 18	10 34	169 26	0 36	298
1	51 20	6 16	8 50	171 10	0 42	416
7/8	44 32	7 46	6 56	173 4	0 53	649
3/4	33 51	10 43	4 43	175 17	1 14	1722
5/8	5 23	25 59	0 40	179 20	3 0	326
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 7°.

The Inclin. to the Equat. is = 84°. [illegible] The Latitude of Vertex is = 84°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
1 1/2	68 54	3 38	15 48	164 12	0 20	123
1 3/8	66 56	4 0	14 17	165 43	0 22	150
1 1/4	64 31	4 28	12 45	167 15	0 24	186
1 1/8	61 30	5 4	11 10	168 50	0 28	239
1	57 36	5 52	9 32	170 28	0 33	323
7/8	52 18	7 3	7 49	172 11	0 40	470
3/4	44 34	9 0	5 57	174 3	0 51	800
5/8	31 22	13 14	3 40	176 20	1 17	1893
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 6°.

The inclin. to the Equat. is = 85°. [illegible] The Latitude of Vertex is = 85°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
1 1/4	68 59	4 20	13 9	166 51	0 21	149
1 1/8	66 34	4 52	11 38	168 22	0 23	190
1	63 28	5 35	10 5	169 55	0 26	251
7/8	59 21	6 34	8 29	171 31	0 31	352
3/4	53 34	8 3	6 48	173 12	0 38	542
5/8	44 36	10 40	4 57	175 3	0 50	1049
1/2	27 14	17 24	2 35	177 25	1 22	1640
*	0 0	0 0	0 0	180 0	0 0	

*·The Course in the Inters. with the Equat. is equal to 5°.

The Inclin. to the Equat. is = 86°. [illegible] The Latitude of Vertex is = 86°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
1	69 3	5 23	10 31	169 29	0 20	191
7/8	65 55	6 15	9 0	171 0	0 23	262
3/4	61 37	7 29	7 26	172 34	0 27	384
5/8	55 16	9 25	5 47	174 13	0 35	642
1/2	44 38	13 6	3 57	176 3	0 49	1572
3/8	18 31	26 8	1 21	178 39	1 37	1114
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 4°.

The Inclin. to the Equat. is = 87°. [illegible] The Latitude of Vertex is = 87°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist
Points	° ′	° ′	° ′	° ′	° ′	Miles
3/4	69 6	7 8	7 53	172 7	0 19	268
5/8	64 41	8 45	6 22	173 38	0 23	420
1/2	57 44	11 25	4 46	175 14	0 30	787
3/8	44 39	17 2	2 58	177 2	0 45	2684
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 3°.

The Inclin. to the Equat. is = 88°. [illegible] The Latitude of Vertex is = 88°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
1/2	69 9	10 36	5 16	174 44	0 16	449
3/8	61 41	14 36	3 43	176 17	0 23	1023
1/4	44 40	24 30	1 58	178 2	0 40	2682
*	0 0	0 0	0 0	180 0	0 0	

* The Course in the Inters. with the Equat. is equal to 2°.

The Inclin. to the Equat. is = 89°. [illegible] The Latitude of Vertex is = 89°.

Course	Lat.	diff.	Long. from the Intrs.	&	diff.	Dist.
Points	° ′	° ′	° ′	° ′	° ′	Miles
1/4	69 10		2 38	177 22		1470
1/8	44 40		0 59	179 1		2681
*	0 0		0 0	180 0		

* The Course in the Inters. with the Equat. is equal to 1°.

(XIV.)

APPLICATION OF THE TABLES No. 51 TO THE SOLUTION OF EXAMPLE 2.

(Ex. 2. Being in the entrance of the English Channel Lat. 49° 6′ N., Long. 6° 2′ W., and bound to New York, Lat. 40° 42′ N., Long. 74° 2′ W. — Page 23, 24, 53 and 55.)

By means of the Inclination of the great circle to the Equator (Lat. of Vertex) = 51° 12′, and the Longitude of the Intersection of that circle with the Equator = 62° 11′ from Greenwich.

(1) *Reduction of all the belonging Elements.*

Spher. Course	Lat.	Long. fr. Greenw.	Dist
73°11′	49° 6′	6° 2′	
Points	° ′	° ′	Mils
6 5/8	49 27	7 48	73
6 3/4	49 46	9 39	74
6 7/8	50 3	11 29	73
7	50 18	13 19	72
7 1/8	50 31	15 9	71
7 1/4	50 42	16 58	70
7 3/8	50 51	18 47	69
7 1/2	50 59	20 35	69
7 5/8	51 4	22 24	69
7 3/4	51 9	24 13	68
7 7/8	51 11	26 1	68
8	51 12	27 49	68
7 7/8	51 11	29 37	68
7 3/4	51 9	31 25	68
7 5/8	51 4	33 14	68
7 1/2	50 59	35 3	69
7 3/8	50 51	36 51	69
7 1/4	50 42	38 40	69
7 1/8	50 31	40 29	70
7	50 18	42 19	71
6 7/8	50 3	44 9	72
6 3/4	49 46	45 59	73
6 5/8	49 27	47 50	74
6 1/2	49 6	49 41	76
6 3/8	48 42	51 33	77
6 1/4	48 17	53 26	79
6 1/8	47 49	55 19	81
6	47 18	57 14	83
5 7/8	46 44	59 9	86
5 3/4	46 7	61 5	88
5 5/8	45 27	63 3	91
5 1/2	44 43	65 2	95
5 3/8	43 56	67 3	99
5 1/4	43 4	69 5	103
5 1/8	42 8	71 10	107
5	41 6	73 17	113
	40 42	74 2	

(2) *Extract of the Stages, Fig. 30, which are Tangents on the Chart.*

Spherical Course	Dist (Mils)	Lat. (° ′)	Long. fr. Greenw. (° ′)
N 73° 11′ W (W b N ¼ N)		49 6	6 2
	73	—	—
W b N ¼ N	74, 73	49 46	9 39
W b N	72, 71	50 18	13 19
W ¾ N	70, 69	50 42	16 58
W ½ N	69, 69	50 59	20 35
W ¼ N	68, 68	51 9	24 13
West	68, 68	51 12	27 49
W ¼ S	68, 68	51 9	31 25
W ½ S	69, 69	50 59	35 3
W ¾ S	69, 70	50 42	38 40
W b S	71, 72	50 18	42 19
W b S ¼ S	73, 74	49 46	45 59
W b S ½ S	76, 77	49 6	49 41
W b S ¾ S	79, 81	48 17	53 26
WSW	83, 86	47 18	57 14
SW b W ¼ W	88, 91	46 7	61 5
SW b W ½ W	95, 99	44 43	65 2
SW b W ¾ W	103, 107	43 4	69 5
SW b W S 56° 46′ W	113, 20, 22	41 6	73 17
		40 42	74 2

(3) *Extract of the Stages, Fig. 31, which are Chords on the Chart.*

Course	Dist (Mils)	Lat. (° ′)	Long. fr. Greenw. (° ′)
N 78° 12′ W		49 6	6 2
W b N ¼ N (N 75° 51′ W)	73, 74	49 27	7 48
W b N (N 78° 43′ W)	73, 72	50 3	11 29
W ¾ N (N 81° 47′ W)	71, 70	50 31	15 9
W ½ N (N 84° 33′ W)	69, 69	50 51	18 47
W ¼ N (N 87° 4′ W)	69, 68	51 4	22 24
West (90°)	68, 68	51 11	26 1
W ¼ S (S 87° 4′ W)	68, 68	51 11	29 37
W ½ S (S 84° 33′ W)	68, 69	51 4	33 14
W ¾ S (S 81° 47′ W)	69, 69	50 51	36 51
W b S (S 78° 43′ W)	70, 71	50 31	40 29
W b S ¼ S (S 75° 51′ W)	72, 73	50 3	44 9
W b S ½ S (S 72° 53′ W)	74, 76	49 27	47 50
W b S ¾ S (S 70° 36′ W)	77, 79	48 42	51 33
WSW (S 67° 23′ W)	81, 83	47 49	55 19
SW b W ¾ W (S 64° 37′ W)	86, 88	46 44	59 9
SW b W ½ W (S 61° 56′ W)	91, 95	45 27	63 3
SW b W ¼ W (S 59° 6′ W)	99, 103	43 56	67 3
S 56° 18′ W	107	42 8	71 10
	155	40 42	74 2

Remarks. 1. The dotted line in table (2) as also in table (3) indicates the limit to which the track passing nearly through Trinity harb. is navigable.

2. The courses expressed in degrees and minutes in table (3) are calculated by Mercator's. Rule by means of latitudes and longitudes of the great circle's arc.

APPLICATION

OF

THE THEORY OF THE GREAT CIRCLE ON THE GLOBE

TO

THE SAILING.

INTRODUCTION.

I.

ON THE CIRCLE TRACED UPON THE SURFACE OF THE EARTH AND THE RHUMB LINE IN GENERAL.

The sphere.

1. *A Sphere* is a solid bounded by one curved surface all the points of which are equally distant from an interior point, which is called the centre of the sphere; it is formed by the revolution of a semicircle about its diameter which remains unmoved.

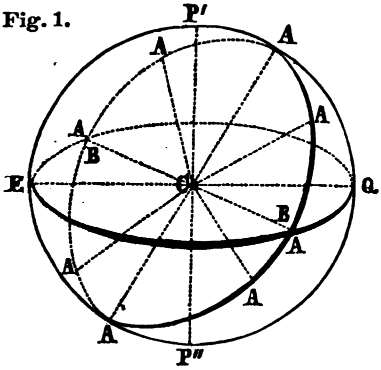
The Radius of a sphere is any straight line drawn from the centre of the sphere to a point of its surface. *All radii of a sphere are equal to one another.*

The Diameter of a sphere is any straight line which passes through the centre, and is terminated both ways by the surface of the sphere. *All diameters of a sphere are equal to one another*, since any one of them is equal to the double of a radius of that sphere.

The circle traced upon the surface of the sphere.

2. The boundary of every section of a sphere made by a plane cutting it is a circle.

Fig. 1.



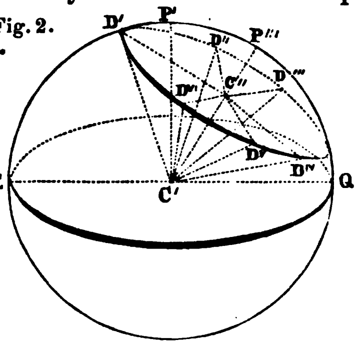
(1) If the cutting plane pass through the centre of the sphere, this is evident; the circle has the radius of the sphere and is called a *Great Circle*.

All Great Circles traced upon the surface of a sphere have the same centre namely that of the sphere, they all are equal to one another, since their radii are equal to each other, and all of them bisect one another, since the plane of any two, *AAA*... and *EBQBE*, intersect in a diameter, $\frac{A}{B}C\frac{B}{A}$, of the sphere. Hence the distance of the two points of intersection, $\frac{A}{B}$ and $\frac{B}{A}$, of two great circles, measured on the sphere's surface, is a semicircumference, a great circle's arc equal to 180°.

Every Great Circle divides the sphere into two equal parts called hemispheres or half globes.

Any two points on the surface of a sphere, except the extremities of the diameter of the sphere, determine the only great circle which passes through them, since the third point for determining the cutting plane is always the centre of the sphere.

Fig. 2.



(2) If the cutting plane does not pass through the centre of the sphere, the boundary of the section is a circle of which the radius is less than the radius of the sphere, and which is called a *Small Circle*.

From C', the centre of the sphere, drop upon the cutting plane the perpendicular $C'C''$, join C'' with any of the points D', D'', D''' &c. of the boundary of the section, and also these points of the boundary with C', the centre of the sphere; then the triangles $C'C''D'$, $C'C''D''$, $C'C''D'''$ &c. are right-angled ones which have the common leg $C'C''$ and hypotenuses $C'D'$, $C'D''$, $C'D'''$ &c. equal to one another, since they are radii of the sphere. Hence $C''D'$, $C''D''$, $C''D'''$ &c., the other legs of those triangles, are equal to one another, and therefore the boundary of the plane section is a circle of which C'' is the centre and $C''D$ the radius. This radius is less than the radius of the sphere, since any leg of a right-angled triangle is always less than the hypotenuse of that triangle.

3. The farther the cutting plane is situated from the centre of the sphere by so much smaller is the circle, because by so much less is its radius. For, (Fig. 2.) in the equation:

$$^{*)}\, C'D'^2 = C'C''^2 + C''D'^2;$$

$C'D'^2$, the square of the sphere's radius is invariable; hence, as $C'C''$, the distance of the cutting plane from the centre of the sphere, increases, the element of the other part of the sum, $C''D'$, which is the radius of the small circle, diminishes.

4. Through any two points A' and A'' on the surface of a sphere an innumerable quantity of small circles may be drawn upon that surface, since the third point for determining the cutting plane is left to one's option. The diameters of those circles differ in their respective lengths, between the length of the straight line $A'A''$ which is always one of the chords of the circles, and a length nearly equal to the diameter of the sphere.

5. If $C'C''$ (Fig. 2.) be produced to meet the surface of the sphere in P''', then P''' is called the *Pole* of the circle $D'D''D'''$.... —

The *Poles* of any circle traced upon the surface of a sphere are always the two extremities of that diameter of the sphere which is perpendicular to the plane of the adopted circle. In Fig. 1. P' and P'' are the poles of the circle EQ, if $P'P''$, a diameter of the sphere, is perpendicular to the plane of the circle EQ.

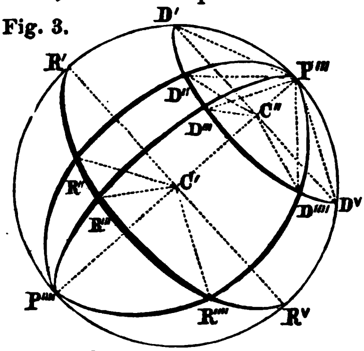
6. Let P''' and P'''' (Fig. 3.) be the poles of the circles $D'D''D'''$..... and $R'R''R'''$...., the planes of which are supposed parallel to each

*) The square of the hypotenuse is equal to the sum of the squares of the legs.

other, then the plane of every great circle passing through P''' and P'''' contains $C''P'''$, $C'P'''$, the perpendicular to the planes of the circles $D'D''$.., and $R'R''$...., therefore all great circles passing through the poles (either of the poles) of a circle, $D'D''$..., $R'R''$...., &c., will cut that circle at right angles.

7. Either pole, P''', of any circle $D'D''$.... is equidistant from every point D', D'', &c. of its circumference, that is, the arcs $P'''D'$, $P'''D''$, &c. are equal to one another.

Fig. 3.



Join C'' with the points D', D'',... then $C''D'$, $C''D''$, &c. are radii of the same circle, and on joining those points, D',D'', ... with either of the poles, P'', the right-angled triangles $P'''C''D'$, $P'''C''D''$, &c. have the common leg $C''P'''$, and their other legs $C''D'$, $C''D''$, &c. are equal to one another. Hence the hypotenuses $P'''D'$, $P'''D''$, &c. of those triangles are equal to one another and therefore also the arcs $P'''D'$, $P'''D''$, &c., since in equal circles equal lines cut off equal arcs.

8. The distances, $P'''R'$, $P'''R''$,...., of either pole, P''', of a great circle, $R'R''$....., from every point in its circumference, measured on great circles, are quadrants, since every radius $C'R'$, $C'R''$, &c., of a great circle, $R'R''$, includes a *right angle* with the radius, $C'P'''$, of the sphere which is perpendicular to the adopted great circle's plane, and every two of those radii, $C'R'$ and $C'P'''$, $C'R''$ and $C'P'''$, &c. lie in the plane of one of the great circles which pass through the pole P'''.

That is to say, the pole of every great circle is 90° from its circumference.

9. The angle at which two arcs of great circles intersect on the surface of a sphere is the angle between the planes in which the arcs lie. But the angle of inclination of these two planes is measured by the rectilinear angle between two radii of the sphere which are perpendicular to the common section of the two planes and of which the one lies in the one plane, and the second in the other.

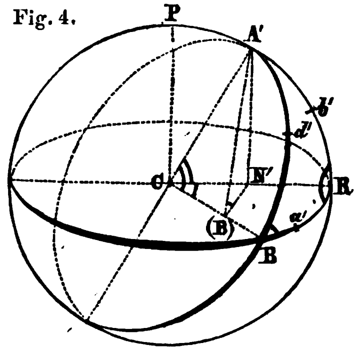
Let the arcs $P'''D''$, and $P'''D'''$ (Fig. 3.) be produced to meet the great circle of which P''' is the pole in R'' and R''', then the angle $R''C'R'''$ and therefore also the great circle's arc $R''R'''$ is the measure of the inclination of the planes in which those arcs are situated and also of the angle at which the arcs $P'''D''$ and $P'''D'''$ intersect.

The Right-angled Spherical Triangle.

10. *A Spherical Triangle* is the portion of the surface of a sphere contained by *three arcs of intersecting great circles.* The *sides* of the triangle are these arcs themselves, the *angles* of it, called *spherical angles*, are the angles between any two of the planes in which those arcs lie.

When in a spherical triangle one of the angles is 90°, that is, when two of the planes of the great circle's arcs which are sides of the triangle are perpendicular the one to the other, the triangle is called *a Right-angled Spherical Triangle.*

Fig. 4.



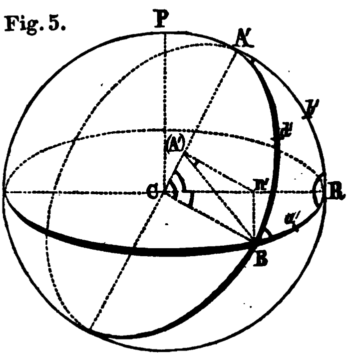
11. Let BR be an arc of a great circle traced upon the surface of a sphere and P one of the poles of that great circle, then the great circle's arc $PA'R$ is perpendicular to BR (Art. 6), and if $A'B$ is an arc of a third great circle, the triangle BRA' is a *right-angled spherical triangle*, the side BA' of which, opposite to $R = 90°$, is the hypotenuse. Farther, draw the radii CA', CB and CR, then the spherical angle B is the angle of inclination of the two planes $A'CB$ and RCB of which CB is the common section.

But, as is known, such an angle of inclination of a plane to a plane is the angle contained by two straight lines drawn from any the same point of their common section at right angles to it, one on one plane, and the other on the other plane.

Now if from A' be drawn $A'N'$ perpendicularly to CR, then this line $A'N'$ drops at the same time perpendicularly from A' upon the plane of the arc BR, and if we imagine another plane which passes through A' and N', this plane will also be perpendicular to the plane of the arc BR and must, in any position by revolving about the line $A'N'$, give a section with the plane RCB which with $N'A'$ includes a right angle.

But revolving that imagined plane about the line $N'A'$ until it be in a position perpendicular to CB, both its sections $(B)N'$ and $(B)A'$ on the planes of the two other sides of the spherical triangle will be perpendicular to CB in the same point (B); hence, the acute angle $A'(B)N'$ opposite to the leg $N'A'$ of the described rectilinear right-angled triangle $(B)N'A'$ gives the measure of the spherical angle B.

Fig. 5.



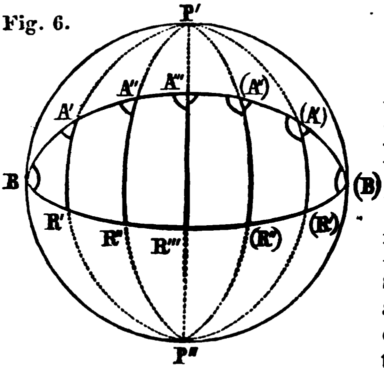
12. Farther, when the line Bn' is drawn perpendicularly from B to CR it will also be perpendicular to the plane CRA', since the planes CRB and CRA' are perpendicular to one another, and an imagined plane which passes through B and n' gives in any position of it, sections right-angled to Bn' on the plane CRA'. Revolving that imagined plane about Bn' until it be in a position perpendicular to the radius CA', the two sections $n'(A')$ and $B(A')$ must be perpendicular to CA' in the same point (A') of it, and therefore the acute angle $B(A')n'$ in the rectilinear right-angled triangle $(A')n'B$ opposite to the leg $n'B$ gives the measure of the spherical angle A'.

13. The object of our investigation, viz. *Great Circle Sailing*, requires but the knowledge of the relations between the sides and angles of the ***right-angled spherical triangle of which one of the angles is an acute one and is invariable.***

Spherical Geometry proves the following relations between the sides and angles of such a triangle.

Let $BR(B)$ (Fig. 6.) be half the circumference of a great circle the poles of which are P' and P'', and $BA(B)$ half the circumference of

Fig. 6.



a second great circle on the surface of the same sphere which has an inclination to the first adopted circle *BR(B)* equal to the acute spherical angle *B*, then the great circles which pass through *P′* and *P″* cut the semicircle *BR(B)* at right angles, and form the right-angled spherical triangles *BR′A′*, *BR″A″*, &c. &c. (one of which has its right angle at *R* and the acute spherical angle *B*) by any two arcs *BA′* and *BR′*, *BA″* and *BR″* &c. &c. being cut off the adopted semicircles, and the third arcs *R′A′*, *R″A″* &c. of the cutting circles themselves. Then we have:

(1) If the sides of the spherical triangle (Fig. 4, 5 and 6.) *BR′A′* are so short in relation to the radius of the sphere, that they might be supposed to be straight lines, the radii of the sphere, *CA′*, *CR* and *CB* (Fig. 4 and 5.) would be nearly parallel to one another, the planes of the two rectilinear triangles *A′N′(B)* (Fig. 4.) and *Bn′(A′)* (Fig. 5.) described in one and the same sphere would lie in one and the same plane, the spherical triangle *BR′A′* would be like a rectangular plane triangle, and the angles *B* and *A* would be complements of each other *).

But in all other cases ***the sum of the two spherical angles B and A is greater than*** 90°.

(2) If *BR‴*, the leg adjacent to the angle *B*, is equal to 90°, then *BA‴*, the hypotenuse, and the spherical angle *A‴* are also equal to 90°. and *R‴A‴*, the leg opposite to the angle *B*, is equal to the measure of that angle, $= B$, since *B* is the pole of the cutting great circle *P′A‴R‴*. (Art. 8, 6 and 9.)

(3) If *BR′*, *BR″*, the leg adjacent to the angle *B*, is less than 90°, then the hypotenuse, *BA′*, *BA″*, and the third angle *A′*, *A″*, are of the same kind as the leg adjacent, as they are also less than 90°, and *R′A′*, *R″A″*, the leg opposite to the angle *B*, is less than the measure of that angle *B;*

if the leg adjacent to the angle *B* increases from zero to 90°, the hypotenuse, the third angle, *A*, and the leg opposite to the angle *B* increase.

(4) If *B(R″)*, *B(R′)*, the leg adjacent to the angle *B* is greater than 90°, then the hypotenuse *B(A″)*, *B(A′)*, and the third angle, *(A″)*, *(A′)*, are of the same kind as the leg adjacent, as they are also greater than 90°, and *(R″)(A″)*, *(R′)(A′)*, the leg opposite to the angle *B* is less than the measure of that angle *B;* and indeed, if $B(R'') = 180^\circ - BR''$, then we have:

$B(A'') = 180^\circ - BA''$; $(A'') = 180^\circ - A''$; and $(R'')(A'') = R''A''$;

if the leg adjacent to the angle *B* increases from 90° to 180°, the hypotenuse and the third angle, *A*, increase, but the leg opposite to the angle *B* diminishes.

*) *For instance.* The edges of a building or of buildings in a certain spot every one of which has the direction of the *plumb-line,* cut one another, if produced downwards, in the centre of the terrestrial globe; but they are parallel to one another for our observation, and the triangle of which the intersections of the sides lie in those vertical lines, equidistant from the centre of the globe, does not differ from a rectilinear plane triangle for our observation, although it is a spherical triangle.

Figure and Dimensions of the Earth.

14. The Earth is, roughly speaking, round, like a ball. More than one proof of this is generally known to seamen: — At the mutual observations of two ships removing from each other under-sail in the open sea, by degrees the hulls will vanish; next the lower sails, till, watching them with a telescope, the upper sails and the top masts vanish altogether. — Sailing northwards or southwards, night after night new constellations of heavenly bodies towards that direction are continually making their appearance, and other constellations towards the contrary direction are sinking lower and lower till they rise no longer. — During an *Eclipse of the Moon,* the shadow of the Earth thrown on the moon is, in all positions of the Earth, circular. — The Earth has been sailed round.

15. Most accomplished mathematicians have made the refined mathematical processes by which the most probable values of the absolute dimensions of the Earth have been obtained from both, the measurements of the terrestrial lines executed with perfect accuracy in different countries, and the most minute determinations of the Azimuths (Bearings) of those lines by observations of heavenly bodies. Lately, Ariy and Bessel have separately arrived by those ways to very nearly the same result, from which is concluded: that the polar diameter of the Earth is shorter than its equatorial by about $\frac{1}{300}$ part.

But the Earth is here, for the investigation of sailing, supposed to be a sphere of which any degree of every great circle on its surface is equal to 60 nautical miles.

Comparison between the circles' arcs and the part of a Rhumb line drawn between two given points on the surface of the Earth.

16. The boundary of a section of the Earth made by a plane cutting it on the surface of the Sea really gives a circle's arc, if we, as is supposed, do not consider the difference between the longest and shortest diameter of the Earth; but on the surface of the Land it would give profiles of hills and mountains, if it were not usual to reduce the places on land to the surface of a globe formed by extending the spherical surface of the sea.

17. Through every place on the Earth's surface an innumerable quantity of great circles may be imagined. The cutting planes which form these great circles contain the plumb line of that place as their common section, wherefore they are without exception „*Vertical Planes*“ *of the place,* and those great circles „*Vertical Circles*“ *of the place.*

18. The Rhumb line is, as is known to seamen, that line upon the surface of the terrestrial globe which has the same inclination to all the *meridians,* as it crosses them in succession.

(1) If this inclination to the meridians is equal to *zero* the rhumb line coincides with a *meridian* and is consequently a great circle.

(2) If that inclination is equal to a *right angle* the rhumb line coincides either with the *Equator* and is in this case also a great circle, or it coincides with a *parallel circle* and is in that case consequently a small circle.

(3) But if finally this inclination of a rhumb line to the meridians is an acute angle, it is a *Spiral**) winding, from its intersection with the Equator, towards one of the poles of the Earth, which it can never reach, since the meridians intersect in the poles and a line which always keeps an inclination to every meridian can never have the direction to either of these points of intersection.

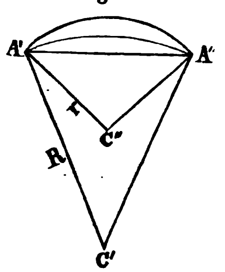
19. *The shortest line which can be drawn between any two places on the surface of the globe is an arc of a great circle passing through them.*

To prove this we have, for our purpose, to take notice only of the arc of the great circle, the arcs of the small circles and the part of the rhumb line making an acute angle with the meridians, drawn between any two points on the surface of a sphere.

In speaking of a circle's arc between two given points which are not the extremities of a diameter we mean the arc which is less than the semicircumference.

(1) Let R be the radius of a sphere, and r the radius of any small circle which passes through the points A' and A'' on the surface of that sphere, then the great circle's arc between these two points will be described with the radius R, greater than r, and both the arcs have the common chord $A'A''$.

Fig. 7.



Now suppose the plane of that small circle to be turned about this straight line $A'A''$, as upon a hinge, and fall upon the plane of the great circle which passes through A' and A'', so as to form with it but one and the same plane, and so that C'', the centre of the small circle adopted, lies on the same side of the common chord $A'A''$ on which the centre of the great circle, C', lies; then it is evident by the flat Fig. 7. that the track on the arc drawn between A' and A'' with the radius R, the radius of the great circle, is shorter than the track on the arc drawn between the same two points with r, the radius of a small circle.

(2) A rhumb line which includes with the meridians an acute angle is a curve of double curvature, wherefore it can not be laid in the plane of the great circle, as in the preceding proof the small circle has been laid; but if both lines, the arc of the great circle and the part of such a rhumb line, are drawn *on the surface of the sphere itself between two given points* every doubt disappears, because the great circle's arc leads directly over the surface and the track on the rhumb line leads as the winding of a screw over the surface from one of the two given points to the other. — Again, we can imagine an arc of a small circle between the two adopted extremities on *the portion* of space comprised between the great circle's arc and the part of the rhumb line, when both do not cross each other; we can imagine such a small circle's arc on either of the *two portions* of this space, when both lines cross each other, and then it is evident, that that part of the rhumb line is greater than that arc of the small circle; hence still greater than the designed arc of the great circle or than the parts of this arc, made by the intersection of the rhumb line and the great circle's arc, when both cross each other.

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*) This kind of the Rhumb line, is mathematically speaking, called „*the loxodromic curve*".

Systems of the circles traced upon the Surface of the terrestrial globe.

20. Let P' and P'' (Fig. 8 and 9.) be the *poles of the Earth*, then

Fig. 8.

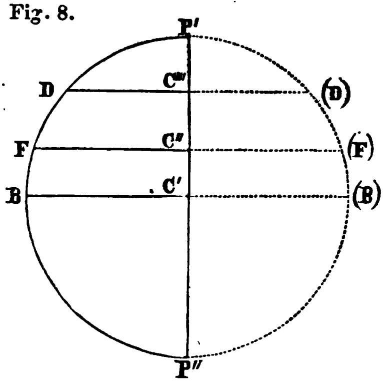
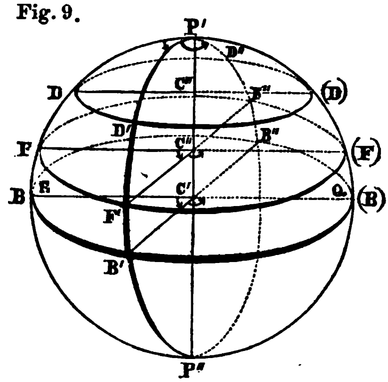


Fig. 9.



the straight line $P'P''$ represents the *Axis of the Earth* and the circle $P'BP''(B)P'$ is a meridian (gives two opposite meridians) of it.

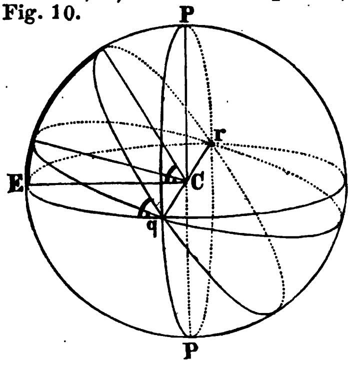
Revolving the semicircle $P'BP''$ (Fig. 9.) about its diameter $P'P''$, the semicircumference $P'BP''$ forms the surface of the terrestrial globe, and every point of it $D, F, B,$ describes the circumference of a circle on that surface. Of these circles only that one which is described by the point B, 90 degrees distant from P', is a great circle, and indeed the *Equator*. All others of these circles, as $FF'F''$...., $DD'D''$...., are small circles, viz. the *Parallel Circles* of the globe. This System of circles of different kinds is generally known by its being traced on charts and maps.

21. Farther, every cutting plane which passes through the poles P' and P'' gives a great circle and here indeed a *Meridian*. The meridians corresponding to equidistant points of the Equator form the known System of great circles which is also generally traced on charts and maps.

22. But for our investigation, we have to arrange all possible great circles which may be drawn on the surface of the globe, and this in a manner which permits us to determine the great circle which passes through two terrestrial places the Long. and Lat. of either of which are given.

For this purpose let q (Fig. 10.) be a point on the Equator, then all great circles which pass through q have a second common intersection, r, with the equator, 180 degrees distant from q, and any one of them is bisected by the Equator into a northern and a southern half, both of which have the same relations to the Equator in all other respects.

Fig. 10.



The position of any one of those great circles is determined by its *Inclination to the Equator* which is measured by one of the equal latitudes of the two points on the great circle, 90° distant from either intersection with the Equator [Art. 9 and Art. 13. (2)]. Any one of these two latitudes is called *Latitude of Vertex*, which is the greatest Latitude of any points of the circle.

A quantity of those great circles which have their common intersection on the Equator, arranged corresponding to equal differences of their inclinations to the Equator, gives a *General System of Great Circles* required.

Such a System includes the Equator itself as its *Base*, has either of the points of the common intersections of their circles as natural *Origin*, and always includes *one of the Meridians*, viz. that of them which passes through the origin of the system.

II.

DESCRIPTION OF THE ACCOMPANYING BLANK CHART CONTAINING A SCALE OF GREAT CIRCLES.

23. Because any point of the Equator may be adopted as the origin of a general system of great circles (Art. 22), the projection of one of them on *the chart on board* is useless. The tracing of a series of such systems the origins of which are supposed equidistant from each other on the Equator would give a crowd of lines that could but spoil the chart altogether.

But that general system of great circles traced on a *Blank Chart shews the positions of the great circles on the globe by their tracks on the chart.*

24. The accompanying *Blank Chart* is a plate separated from the book extended to 180° of longitude and 70° of latitude on one side of the equator; hence it represents half the northern and also half the southern hemisphere embracing larger latitudes than Great Circle Sailing may require.

It contains in Mercator's projection: 1. *the meridians* drawn through each fifth degree of longitude; 2. *the parallels* drawn through each fifth degree of latitude; and 3. the *Arcs of the Great Circles* of a general system of such, drawn to each degree of Inclination to the equator or, which is the same, to each degree of Lat. of Vertex.

The longitudes reckoned from either of the common intersections of the circles and the latitudes are designed with numbers in the usual manner. Of the curves which design the tracks of the great circles, each fifth is distinguished by being darker than the others and is marked with a *Number* which indicates its inclination to the equator (its Lat. of Vertex) in whole degrees.

On those tracks, marked with numbers, the Courses corresponding to each quarter of Compass Points are designed by short parts of the meridians in which the courses take those dimensions. Any one of these Courses to be expressed in whole Points is designed with a number equal to that quantity of Points; any one of them which contains a half of a Point is only designed with the fraction $\frac{1}{2}$, and any one of them which contains one quarter or three quarters of a Point is without other marks than the said part of the meridians in which the course has one of those magnitudes.

The numbers of always four circles, drawn between two successive circles designed with numbers, are omitted, as well as the marks of the Courses on those tracks, not to crowd the Blank Chart with numbers and marks.

The tracks of the intermediate circles between the drawn ones may be imagined, or drawn by hand, with sufficient accuracy on the Blank Chart, considering the scale adopted for its construction.

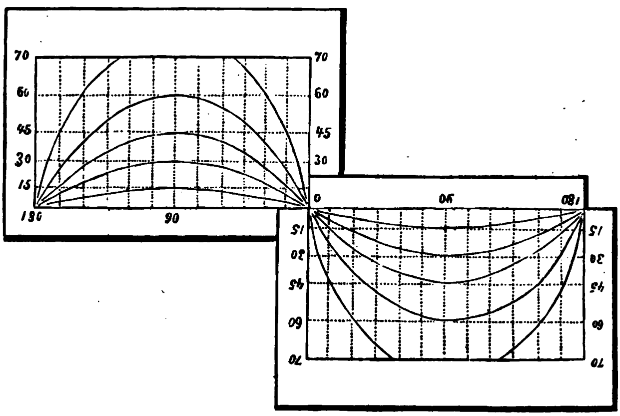
25. For shewing the track on a great circle the extremities of which are on the same side of the equator and the producing of such a trak to the equator, one plate of the Blank Chart is sufficient, since the origin of the drawn system may be any one of the points of the Equator, hence also that point in which the track produced intersects the equator.

26. Also one plate of the Blank Chart will be sufficient to determine the arc of a great circle which passes through two given places, by the intersection of that circle with the Equator and the inclination (Lat. of Vertex) to it, whether the two given places be on the *same* side of the Equator or on *opposite* sides. (Art. 31—37.)

27. But for a general survey of the track on a great circle, the extremities of which are on opposite sides of the equator, two plates of the Blank Chart are advantageous. In one of these plates the corners of the paper, contained between the produced lines of the equator and the meridians of the two origins, are to be cut out.

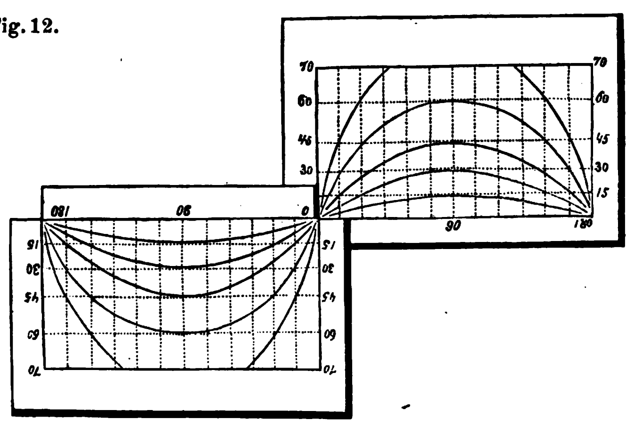
Then, if the track leads eastward from the northern hemisphere to the southern, or westward from the southern to the northern hemisphere, the two plates are to be placed together in the way shewn by Fig. 11.

Fig. 11.



But if the track leads westward from the northern hemisphere to the southern, or eastward from the southern to the northern hemisphere, the two plates are to be put in the position which Fig. 12 shews.

Fig. 12.



III.

DETERMINATION OF THE GREAT CIRCLE WHICH PASSES THROUGH TWO GIVEN PLACES.

Remarks in respect to the Solution.

28. The Intersection of a great circle with the Equator determines the *general System of Great Circles,* which includes the circle, and the Inclination of the circle to the Equator determines the position of the circle in that system. Hence the position of a great circle will be completely determined by having found both, its Intersection with the Equator and its Inclination to the Equator. (Art. 22.)

29. The intersection of a great circle with the Equator is determined by the two kinds of longitude of any place of the circle, one from the first meridian (geographical longitude) and the other from that intersection, because the longitude of that intersection itself from the first meridian is always equal to the difference of those two kinds of longitude of any one and the same place on the circle.

30. The different kinds of solution of the problem, to find that Intersection and Inclination (Art. 28 & 31) of a great circle which passes through two given places, having given both latitudes and Diff. Long., may be reduced in both cases, namely

(1) the places are on the same side of the Equator, or
(2) they are on opposite sides,

to either of them:

either, by retaining the given magnitudes and their positions;

or, by retaining only the magnitudes of the given latitudes, but substituting for the given Diff. Long. the *Supplement of it* and supposing, at the same time, the places to be on the same side of the equator, when they are on opposite sides, and on opposite sides, when they are on the same side, both by transferring one of the places to the other side of the equator.

Fig. 13.

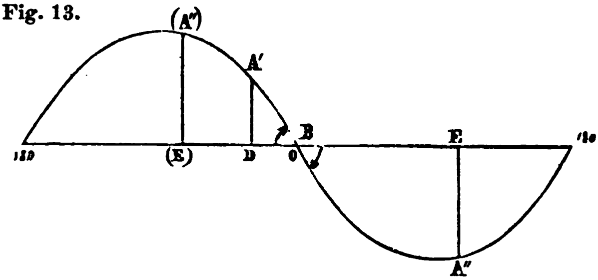
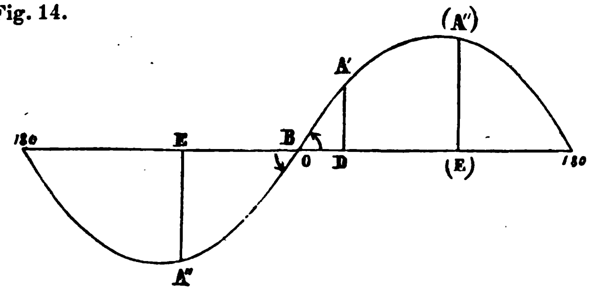


Fig. 14.



Let A' & A'' (Fig. 13 & 14) be two places *on opposite sides of the equator* and the curve $A''BA'(A'')$ the track of the great circle which passes through A' & A'' in Mercators projection, then DA' represents the Lat. of A', EA'' that of A'' and DE the given Diff. Long.

Now, if $E(E)$ is supposed equal to 180° and $(E)(A'')$ is drawn perpendicularly to the equator, EA'' & $(E)(A'')$ are on the same meridian (on two opposite meridians, if the meridian is adopted as semicircle the extremities of which are the poles of the Earth), $(E)(A'')$, the latitude of (A''), is equal to EA'', the latitude of A'' and $D(E)$, the Diff. Long. of A' and (A'') is the supplement of DE, the given Diff. Long.; but A' and (A'') are *on the same side of the equator.*

But if A' & (A'') be two given places *on the same* side of the Equator, we have

by transferring $(E)(A')$ from (E) to E,
of the given Diff. Long. $D(E)$, but A' ...

CHIEF

31. *To find the longitud...*
...ircle with the Equator, a' & ...
...ircle to the Equator...
...atitudes, b' & b'', ...
...hrough which ...

ERRATUM.

For „in the direction from b' to b''“ page 45, 2 line, ***substitude and read:*** —
(in the direction from b' to b'', if the base a exceeds 90°, or the Index poin[t] of b' indicates a higher curve than tha[t] point of b''; but in the direction from b' to b', if the Index point of b' indicate[s] a lower curve than that point of b'')

...Y INSPECTION,

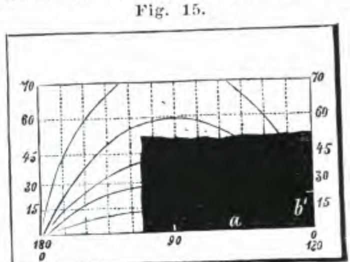
...reat Circles on the accompanying Blank Chart.

...the Blank Chart is to be reduced to the case in which the places ...quator. If the given places are on opposite sides, one of them is to ...pposed as transferred to the other side. Art. 30.)

To make the „Index Model.“

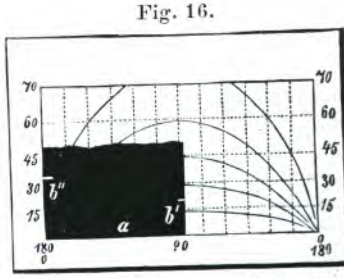
... off two of the sides of a piece of thick paper to straight-...es right-angled to each other. Place one of these edges along ...ator line, the other along the meridian of one of the common in-...ons of the curves. We have placed ... right hand side in Fig. 15. In this ...n of the paper, mark the extremity ... from the scale of longitudes on the ...acent edge, as *Base of the Model*, ...ual to Diff. Long., if the given places ...re on the same side of the Equator, but equal to the supplement of Diff. Long., if they are on opposite sides; and mark on the other edge one of the given latitudes from the adjacent scale of latitudes, as one of the Index lines, here b', the less. latitude.

Fig. 15.



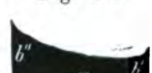
In the mark made as extremity of the base, a, cut off the paper right-angled to a (parallel to b'), and place a along the equator and the new straight edge along the meridian of the other common intersection, here along that to the left hand; viz. Fig. 16. In this position of the paper mark on that new edge b'', the other latitude, from the adjacent scale of latitudes as second Index line.

Fig. 16.



The extremities of b' and b'' we shall call „Index points“. Between these two, finally cut off the paper in such a manner that they be represented as points of acute-angled corners, as Fig. 17, shews.

Fig. 17.

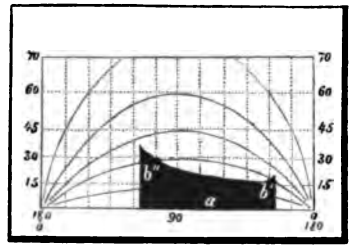


To dispose the Index Model's edges as Indices upon the Blank Chart.

?3. Place a, the base of the Model along the equator and b'', the higher latitude, along the meridian of 90° of longitude: If in this position both Index points (the extremities of b'' and b') hit one and the same curve or lie symmetrically between two and the same curves succeeding each other, the position is right. In all. other cases

slide the Model with its base, *a*, along the equator line, in the direction from b' to b'', till it gets one of the two positions just now described, viz. Fig. 18. considering that in this figure all the curves of the Scale of Great Circles could not be represented.

Fig. 18.



34. In this position of the Model either of the Index lines shews directly on the scale of longitudes, ***the longitudes from the intersection of the required great circle with the Equator*** of either of the given places, if they are on the same side of the Equator, but if they are on opposite sides, that Index line which represents the transferred latitude shews the supplement of the longitude to be found, reckoning this longitude from the intersection in the right position; but it shews the longitude itself of the transferred place reckoning from the opposite intersection as origin.

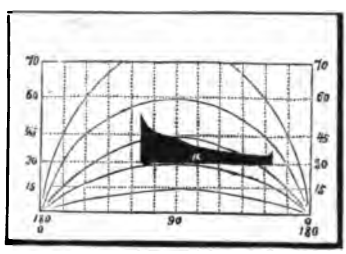
35. ***The Inclination of the great circle to the Equator*** (Lat. of Vertex) is also indicated by the right position of the Model upon the Blank Chart. If the Index points lie on one and the same curve that inclination is in whole degrees equal to the Number of this curve. If those extremities of b' and b'' lie in the middle of the interval of two and the same curves succeeding each other, the Number of the lower curve of the two, gives the whole degrees of the inclination to be found, to which a half degree (30 minutes) is to be added. If the Index points lie on a third from the lower curve, a third degree (20 minutes), if they lie on a quarter from the lower curve, a quarter of a degree (15 minutes), &c. &c. is to be added to the quantity of whole degrees determined by the Number of the lower of the two curves which limit the divided interval.

Separate case.

36. If both places are in high latitudes the Model may be formed for sliding it along one of the printed Parallels instead of sliding it along the equator line; viz. Fig. 19.

In this case the extremities of the latitudes are to be marked by a position of the paper for making the Model with its base, *a*, on the Parallel adopted as the sliding line.

Fig. 19.



For determining the longitudes by such a Model mark the Index points on the Blank Chart itself, after having fixed those points in the right position upon it, and lay the edge of a ruler so that it pass through such a mark and cross equally both scales of longitudes. Thus, that edge of the ruler gives the Index line for one of the longitudes required.

All other instructions remain as before said.

37. To judge of the position of any one of the Index points within the interval of two neighbouring curves, of the lines which can be imagined as passing through that point and terminated both ways by those curves; either that line which drops at right angles upon both the curves, or the part of the meridian, if it is smaller than

…ring *(E) (A')* from *(E)* to *E*, … Diff. Long. *D(E)*, but *A'* &…

CHIEF…

To find the longitud… …ith the Equator. … to the Equato… …des, b' & b''… …gh which…

ERRATUM.

For „in the direction from *b'* … page 45, 2 line, *substitude and re…*
(in the direction from *b'* to *b''*, … base *a* exceeds 90°, or the Index … of *b'* indicates a higher curve tha… point of *b''*; but in the direction fr… to *b'*, if the Index point of *b'* ind… a lower curve than that point of…

…y INSPECTION,

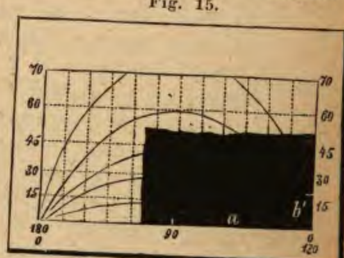
…Great Circles on the accompanying Blank Chart.

(… the Blank Chart is to be reduced to the case in which the places … equator. If the given places are on opposite sides, one of them is to … supposed as transferred to the other side. Art. 30.)

To make the „Index Model."

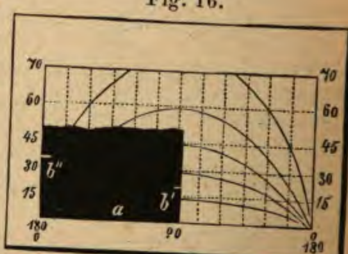
…Cut off two of the sides of a piece of thick paper to straight-…dges right-angled to each other. Place one of these edges along …uator line, the other along the meridian of one of the common in-…ctions of the curves. We have placed … the right hand side in Fig. 15. In this …sition of the paper, mark the extremity …f *a* from the scale of longitudes on the adjacent edge, as *Base of the Model*, equal to Diff. Long., if the given places are on the same side of the Equator, but equal to the supplement of Diff. Long., if they are on opposite sides; and mark on the other edge one of the given latitudes from the adjacent scale of latitudes, as one of the Index lines, here *b'*, the less. latitude.

Fig. 15.



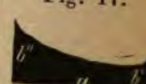
In the mark made as extremity of the base, *a*, cut off the paper right-angled to *a* (parallel to *b'*), and place *a* along the equator and the new straight edge along the meridian of the other common intersection, here along that to the left hand; viz. Fig. 16. In this position of the paper mark on that new edge *b''*, the other latitude, from the adjacent scale of latitudes as second Index line.

Fig. 16.



The extremities of *b'* and *b''* we shall call „Index points". Between these two, finally cut off the paper in such a manner that they be represented as points of acute-angled corners, as Fig. 17, shews.

Fig. 17.

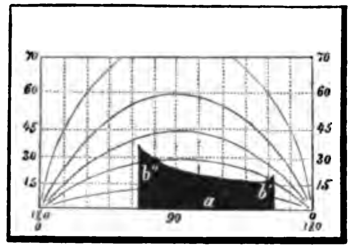


To dispose the Index Model's edges as Indices upon the Blank Chart.

23. Place *a*, the base of the Model along the equator and *b''*, the higher latitude, along the meridian of 90° of longitude: If in this position both Index points (the extremities of *b''* and *b'*) hit one and the same curve or lie symmetrically between two and the same curves succeeding each other, the position is right. In all. other cases

slide the Model with its base, *a*, along the equator line, in the direction from b' to b'', till it gets one of the two positions just now described, viz. Fig. 18. considering that in this figure all the curves of the Scale of Great Circles could not be represented.

Fig. 18.



34. In this position of the Model either of the Index lines shews directly on the scale of longitudes, ***the longitudes from the intersection of the required great circle with the Equator*** of either of the given places, if they are on the same side of the Equator, but if they are on opposite sides, that Index line which represents the transferred latitude shews the supplement of the longitude to be found, reckoning this longitude from the intersection in the right position; but it shews the longitude itself of the transferred place reckoning from the opposite intersection as origin.

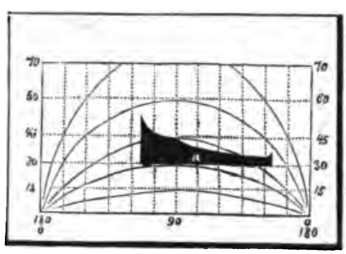
35. ***The Inclination of the great circle to the Equator*** (Lat. of Vertex) is also indicated by the right position of the Model upon the Blank Chart. If the Index points lie on one and the same curve that inclination is in whole degrees equal to the Number of this curve. If those extremities of b' and b'' lie in the middle of the interval of two and the same curves succeeding each other, the Number of the lower curve of the two, gives the whole degrees of the inclination to be found, to which a half degree (30 minutes) is to be added. If the Index points lie on a third from the lower curve, a third degree (20 minutes), if they lie on a quarter from the lower curve, a quarter of a degree (15 minutes), &c. &c. is to be added to the quantity of whole degrees determined by the Number of the lower of the two curves which limit the divided interval.

Separate case.

36. If both places are in high latitudes the Model may be formed for sliding it along one of the printed Parallels instead of sliding it along the equator line; viz. Fig. 19.

In this case the extremities of the latitudes are to be marked by a position of the paper for making the Model with its base, *a*, on the Parallel adopted as the sliding line.

Fig. 19.



For determining the longitudes by such a Model mark the Index points on the Blank Chart itself, after having fixed those points in the right position upon it, and lay the edge of a ruler so that it pass through such a mark and cross equally both scales of longitudes. Thus, that edge of the ruler gives the Index line for one of the longitudes required.

All other instructions remain as before said.

37. To judge of the position of any one of the Index points within the interval of two neighbouring curves, of the lines which can be imagined as passing through that point and terminated both ways by those curves; either that line which drops at right angles upon both the curves, or the part of the meridian, if it is smaller than

the part of the Parallel, but instead of this part of the meridian the part of the Parallel, if it is the smaller of the two, is to be divided by the eye into equal parts. Such a division and the comparison between those two, for examining the symmetrical position required, are to be executed exactly by the eye, because the intervals between any two neighbouring curves of the Blank Chart are always sufficiently small for either of them to be taken in at first sight.

After a trial or two the position of the Model will be rightly fixed, but the coinciding of its base with the sliding line must always be strictly attended to.

By Construction.

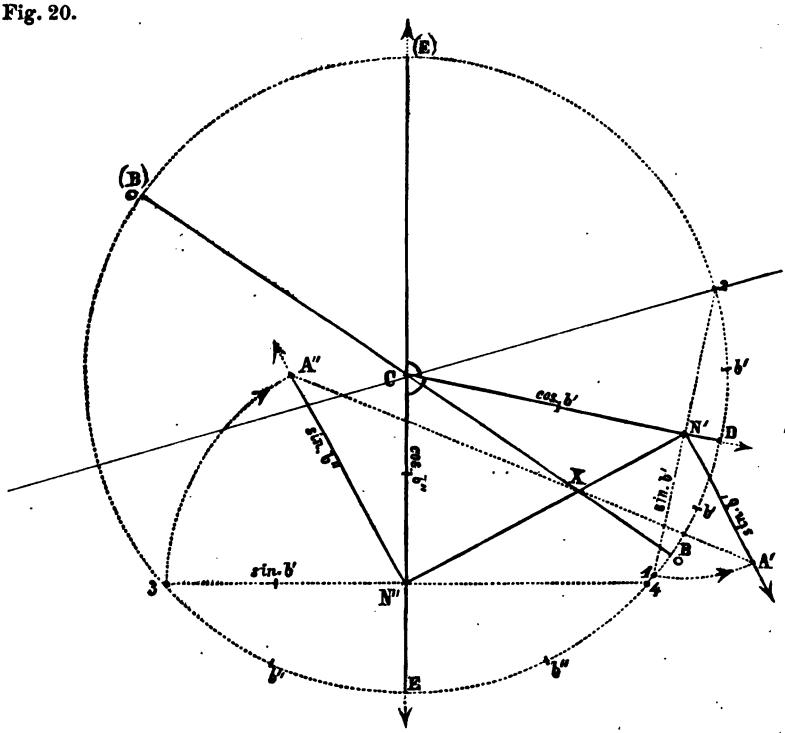
(The Solution by construction is to be reduced to the case in which the places are on opposite sides of the equator. If the given places are on the same side, one of them is to be supposed as transferred to the other side. Art. 30.)

(1) *To find the Intersection of the great circle with the Equator and the longitudes from it.*

(Proof of the Construction: Appendix IV, 3, Fig. 9.)

38. In Fig. 20 the arc *DE* is the Diff. Long., if the places are on opposite sides of the Equator, it is the supplement of Diff. Long. if they are on the same side;

Fig. 20.



the arcs $D1 = D2 = b'$ are equal to one of the latitudes, here to the less; the arcs $E3 = E4 = b''$ are equal to the other given latitude; $N'A'$ & $N''A''$ are perpendicular to $N'N''$, and $N'A'$ is equal to $N'1$, $N''A''$ equal to $N''3$; X, the point found by the construction, is the determining point for the diameter $(B)\,CB$, of which the extremities, (B) and B, are the *Intersections required*; the arc BD is the *longitude from the intersection* of the place in lower latitude and the arc BE *that longitude* of the second given place, if the places are on opposite sides of the Equator, but if they are on the same side, that arc of the two which is determined by the latitude of the transferred place indicates the supplement of the longitude of this place.

39. (The straight line $A'A''$ is the chord of the ***spherical distance*** between the given places, if they are on opposite sides of the Equator, in the other case, it is the chord of the supplement of that distance.)

By means of the Scale of Chords Fig. 20.

40. With the chord of 60° describe a circle; draw a diameter, here *(E)E,* and mark the centre *C.*

Lay off ***ED*** equal to ***Diff. Long.***, if the places are on opposite sides of the Equator, but equal to the ***supplement of Diff. Long.***, if they are on the same side; and draw the radius ***CD.***

Lay off ***D1*** & ***D2*** either of them equal to the latitude of the place on the right hand, here equal to b', the smaller latitude. Lay off ***E3*** & ***E4***, any one of them equal to the other latitude, here equal to b'', the larger of the given latitudes.

Draw the chords ***12*** & ***34***. Now join $N'N''$. Draw $N'A'$ and $N''A''$, both perpendicularly *) to $N'N''$, but on opposite sides of it.

Set off by the compasses $N'1$ from N' upon $N'A'$, and $N''3$ from N'' upon $N''A''$. Join $A'A''$, which line crosses $N'N''$ in the point ***X.*** Finally draw ***CX*** meeting the circumference in the point $_oB$, then the arcs ***BD*** and ***BE*** represent the respective longitudes required, **or**, in case one of the places be transferred to the other side of the equator for making the construction, the longitude of this place by the supplement of it.

By means of the Protractor Fig. 20.

41. Draw a straight line, in Fig. 20 piinted red, and mark on it the centre *C.* The notch on the lower edge of the protractor which designs its centre is to be placed on *C* and that edge to be placed along the red line. Where this line comes out from beneath the protractor the point ***2*** is to be marked. Next, the following arcs are to be pricked off from the scale of the protractor: ***2D***, equal to one of the latitudes, here equal to b', the less latitude; ***21***, equal to the double of that latitude; $2E = (b' + a)$ equal to the sum of the first adopted latitude and ***Diff. Long.*** (resp. ***the supplement of Diff. Long.***); $24 = (b' + a - b'')$ equal to the difference between that sum and the other latitude; and $23 = (b' + a + b'')$, equal to the sum of ***Diff. Long.*** (resp. ***the supplement of Diff. Long.***) and both latitudes.

Draw the radii ***CD*** and ***CE***, and also the chords ***12*** and ***34***, (to do so, the circumference of the circle is not to be described by the compasses, because the points of it which are to be asked for are pricked off.); and finish the construction as just before said. (Art. 40.)

Controls of the construction.

42. (1) Producing $N'A'$ beyond N', and $N''A''$ beyond N'', and making these prolongations, respectively, equal to $N'A'$ & $N''A''$, then the straight line between these new extremities must pass through the point X.

(2) Repeating the construction, Art. 40 or Art. 41, on the opposite side of the adopted base (*E(E)* Art. 40; the red line Art. 41) beginning at the opposite extremity of it, the edge of a ruler placed on likenamed points must pass through the point C; any two likenamed chords must be parallel to one another; the two lines between the intersections of a radius and a chord must be parallel to each other; also the perpendiculars (the parallels) on the extremities of these two lines must be parallel to one another; finally the last constructed point X' must be a point of the diameter *B(B).*

*) If the distance (Art. 39) is not required, $N''A''$ may be drawn perpendicularly to $N''N'$ by the eye, but then $N'A'$ exactly parallel to $N''A''$.

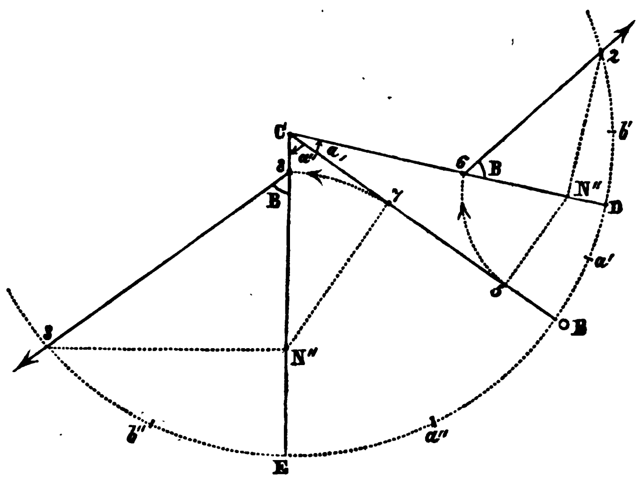
(2) *To find the Inclination of the Great Circle to the Equator (Lat. of Vertex).*

(Proof of the Construction: Appendix II. Fig. 3)

By means of the construction of the Intersection of the great circle with the Equator. Art. 38.

43. In Fig. 21 the points *2*, D, OB, E and *3* on the part of the printed circumference the centre of which is C, are the same as the points so denominated in Fig. 20. Also the points N' and N'' are the same as those points in Fig. 20. Newly designed are in Fig. 21, the longitudes from the intersection as angles and arcs and indeed with a' this longitude of the place A', that in the smaller latitude, with a'' this longitude of the place A'', that in the higher latitude, if the places are on opposite sides of the Equator; but if they are on the same side, one of the two, a' or a'', that one of them which belongs to the transferred place designs the supplement of the longitude required. The new construction embraces the perpendiculars $N'5$ & $N''7$, the lengths $N'6$ & $N''8$, respectively, equal to those perpendiculars, and the lines *62* & *83*. The constructed angles $D62$ and $E83$ are equal to each other and equal to the required angle B, the Inclination of the great circle to the equator.

Fig. 21.



44. From N' drop the perpendicular $N'5$ upon CB, set off by the compasses $N'5$ from N' upon $N'C$ which determines the point *6*, and join *62*, then the angle $D62 = B$ is equal to the Inclination required, Lat. of Vertex;

or:

from N'' drop the perpendicular $N''7$ upon CB, set off by the compasses $N''7$ from N'' upon C which determines the point *8*, and join *83*, then the angle $E83 = B$ is equal to the Intersection required.

Controls of the construction.

45. (1) If both the angles $D62$ and $E83$ are constructed, they must be equal to each other.

(2) Produce $N'5$ beyond *5* until it cuts the circumference, then the distance from *5* to this point of intersection, not printed in the figure, must be equal to the hypotenuse *62*; also, if we produce $N''7$ beyond *7* till it meets the circumference, the distance from *7* to the point of intersection and the hypotenuse *83* must be equal to each other.

By Calculation.

(1) *To find the longitudes from the Intersection of the great circle with the Equator.*

(Proof of the Formula: Appendix III.)

46. For both cases, whether the places be on the same side of

the Equator or on opposite sides, taking the sums and differences algebraically, we have the same

Rule.

Take half of D. Long. Take the sum of the lats. and their diff.

Add together the log. tan. of half the D. Long., the log. sine of the sum of the lats. and the log. cosec. of their diff.; the sum (rejecting twenty) is the log. tan. of half the sum of the two required longitudes.

The ***sum****) of the half sum of these two longitudes and the half D. Long. is the longitude from the intersection **) of the great circle with the Equator of the place A'', the place in the ***higher*** of the two latitudes; the ***difference****) of the said half sum and the half D. Long. is the other of the required longitudes, that of the place A', the place in the ***smaller*** of the two latitudes.

[Ex. 1, 1; page 22, (X). — Ex. 2, 1; page 23, (XI). — Ex. 3, 1, page 24, (XII).]

(2) *To find the Inclination of the Great Circle to the Equator (Lat. of Vertex).*

[Solution of the right-angled spherical triangle; Appendix II, formula (4).]

Rule.

47. Find the Long. of either of the places from the Intersection of the great circle with the Equator (Art. 46); then to its log. sine add the log. cotan. of the latitude of the same place; the sum (rejecting ten) is the log. cotan. of the required Inclination (Lat. of Vertex).

[Ex. 1, 2; page 22, (X). — Ex. 2, 2; page 23, (XI). — Ex. 3, 2; page 24, (XII).]

(3) *To find the geographical Long. of the Intersection of the great circle with the Equator.*

By means of the same place's geographical longitude and longitude from the Intersection of the great circle with the Equator. (Art. 29 and 46.)

Rule.

48. Subtract algebraically from the geograph. Long. of the determined place its long. from the said Intersection (Table's long.); the difference is the required geographical longitude of that Intersection.

[Ex. 1. 3; page 22, (X). — Ex. 2, 3; page 23, (XI). — Ex. 3, 3; page 24, (XII).]

Controls of the calculation.

49. (1) If the rule Art. 47 is applied twice, employing successively the two given places, the result must give the same value.

[Ex. 1, 2; page 22, (X). — Ex. 2, 2; page 23, (XI). — Ex. 3, 2; page 24, (XII).]

***) With the exception, in case the D. Long. exceeds 180° and is kept in applying the rule, as has been done in Ex. 3, 1; page 24, (XII). In this case the said *sum* gives the required longitude of the place in the *smaller* latitude and the said *difference* that longitude of the other place, the one in the *higher* latitude. But if in that Ex. 3, the supplement to 360° were taken for D. Long., we should have D. Long. = 129° 11′; half D. Long. 66° 35½′, and then directly by applying the rule,**

P. Jack. L. fr. the Int. (sum) 117° 7′ W.

Panama L. fr. the Int. (diff.) 12 4 E.

which values are obtained pag. 24, by taking the supplement of the longitudes there found by the rule.

****) The longitudes found are reckoned from that one of the two points of intersection of the great circle with the equator which is the *nearer* to the meridian of the place in the *smaller* latitude; with the exception of the case in which the D. Long. exceeds 180° and is kept in applying the rule, as is done in Ex. 3, 1; page 24 (XII). In this case the longitudes so found are reckoned from that one of the said intersections which is the *nearer* to the place in the *higher* latitude.**

When the longitudes found by the rule are to be reckoned from the other point of those intersections, their supplements must be taken, changing their names at the same time (W into E and E tion W); as in Ex. 2, 1; page 23, (XI) and Ex. 3, 1; page 24, (XII). — (Sketches pag. 25 and Art. 50.)

(2) Applying the rule Art. 48 twice, by means of each of the two places separately, the values of the results must also be equal to each other; but this proof is not sufficient, it is only a proof of a certain part of the calculation. Wherefore that rule is made the third, although it might have been applied directly after the first.

Sketch to shew the Position of the Two Points of Intersection of the Great Circle with the Equator.

50. After having found, by any one of the given methods of solution, the longitudes of the given places reckoned from either of those points of intersection, the position of these two points referring them to the first meridian (meridian of Greenwich) must be imagined clearly. For this purpose a sketch of the position of those points will be convenient. We propose to make it by hand in the following manner.

To represent the equator line draw a straight line. Imagine or mark on it a quantity of equal small parts any one of them adopted as 10° of the scale of longitudes. Mark on this line the extremities of D. Long. (when it exceeds 180°, its supplement to 360°). Set off perpendicularly to the equator line in these extremities the two given latitudes in their right position and indicate the places themselves.

From each of the feet of these two latitudes set off: —

1. the respective longitudes from the intersection of the great circle, in the contrary direction of their names (E. if it is W.; W. if it is E.), which give one and the same point, that point of intersection from which the longitudes found by the solution are reckoned; and the supplements of those longitudes in the direction of each longitude's name itself, (E. if it is E., W. if it is W.), which give the other point of intersection of the great circle with the Equator;
2. respectively, the given geographical longitudes in the directions contrary to their own, (E. if it is W., W. if it is E.) which give the origin of the geographical longitudes, the point in which the first meridian (Meridian of Greenwich) crosses the Equator.

Set off perpendicularly to the equator line the Lat. of Vertex in the middle between the two points of Intersection of the great circle with the Equator (of always two of them).

Finally draw a curved line passing through the two places, through the Vertex of the circle's track and through the points of intersection with the equator, to represent the track of this circle (semicircle).

Fig. 22 is such a sketch of the great circle which passes through St. Helena and Cape Horn. To draw it, we have *laid off* in the following succession:

D. Long.	=	61°,	
Lat. of C. Horn	=	56 S.,	
Lat. of St. Helena	=	16 S.,	
C. Horn's long. from the Inters.	=	72 W.	eastwards,
its supplement	=	108 —	westwards,
St. Helena's long. from the Inters.	=	11 W.	eastwards,
its supplement	=	169 —	westwards,
Lat. of Vertex	=	57 S.,	
C. Horn's geographical long.	=	67 W.	eastwards,
St. Helena's geographical long.	=	6 W.	eastwards.

Fig. 22.

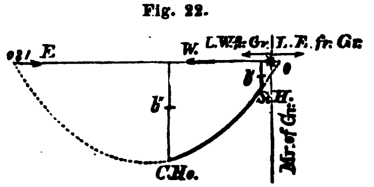


Fig. 23 is a sketch of the great circle which passes through a place in the entrance of the English Channel, Lat. 49°6′ N., Long. 6°2′ W. and New York. To draw it, we have *laid off* in the following succession:

D. Long.	=	68°,	
lat. of New York	=	41 N.,	
lat. of the adopted place	=	49 N.,	
New York's long. from the Inters.	=	136 W.	eastwards,
its supplement	=	44 —	westwards,
the adopted place's long. from the Inters.	=	68 W.	eastwards,
its supplement	=	112 —	westwards,
Lat. of Vertex	=	51 N.,	
New York's geographical long.	=	74 W.	eastwards,
the adopted place's geographical long.	=	8 W.	eastwards.

Fig. 23.

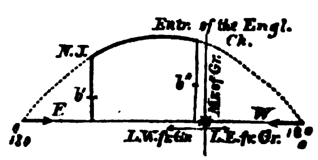
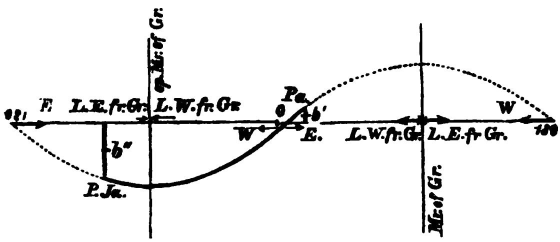


Fig. 24 is a sketch of the great circle which passes through Panama and Port Jackson.

Fig. 24.



To draw it, we have *laid off* in the following succession:

the supplement to 360° of D. Long.	=	129°,
lat. of Port Jackson	=	34 S.,
lat. of Panama	=	9 N.,
P. Jackson's long. from the Inters.	=	117 W. eastwards,
its supplement	=	63 *) westwards,
Panama's long. from the Inters.	=	12 E. westwards,
its supplement	=	168 — eastwards,
Lat. of Vertex	=	37 N. and S.
P. Jackson's geographical long. . . .	=	151 E. westwards, in two parts 63° *) and 88°,
Panama's geographical long.	=	80 W. eastwards.

IV.

THE SPHERICAL COURSES AND THE SPHERICAL DISTANCES ON A DETERMINED GREAT CIRCLE'S ARC.

The Spherical Course being adopted.

51. The solution of the three problems:

having given the inclination of the great circle to the Equator and any spherical Course on the circle, to find 1. the latitude, — 2. the longitude from the intersection of the great circle with the Equator, — and 3. the distance from that intersection — of that point of the circle at which the given Course takes place;

is applied in forming the accompanying Tables to facilitate the practice of great circle sailing. The respective formulæ are given in the *Description* of these Tables [Page 10 (III)], and proved: Appendix, II, respectively (8), (9) and (10).

Employing those Tables, the said problems will be superfluous as also by employing the *Scale of Great Circles* on the Blank Chart. (Art. 24.)

Determination of the Spherical Course.

52. Besides these spherical Courses determined by the accompanying Tables and Blank Chart, only the one at the beginning of the ship's track may be asked for in practice, which leads to

Problem II.

To find the spherical Course at the ship's place, after having determined the great circle which passes through that place and the place of the ship's destination. (Chief Problem Art. 31.*)*

By Inspection,

by means of the Scale of Great Circles on the accompanying Blank Chart.

53. After having brought the „Index Model" into the right position upon the Blank Chart (Art. 33 & 37), mark the ship's place on

the Blank Chart itself by the respective Index point. Draw on it with a lead-pencil a certain part of the determined great circle's track (Art. 24) both ways of that point to equal length. Place the centre of the horn protractor on the said point and its diameter both ways from the centre symmetrical to the branches of the drawn curve. Hold the thread of the protractor parallel to the meridians; the required spherical course will then be read off on the graduated edge.

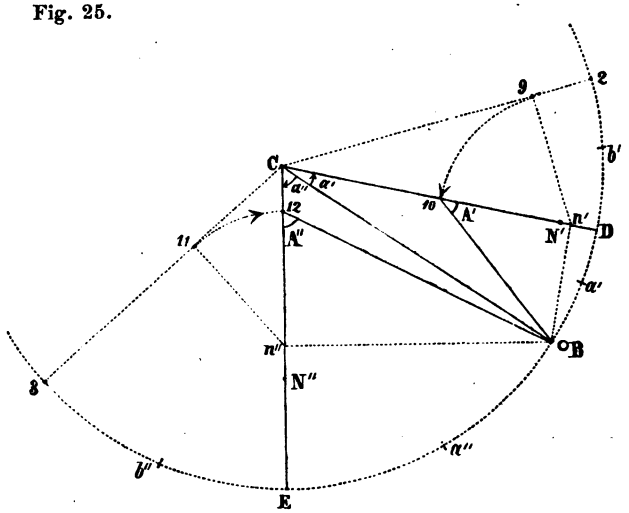
By Construction.

(Proof of the construction: Appendix II, Fig. 4.)

By means of the construction to find the Intersection of the great circle with the Equator. Art. 38.

54. In Fig. 25 the points *2*, *D*, o*B*, *E*, and *3* on the printed part of the circumference the centre of which is *C*, are the same as the points so denominated in Fig. 20. Also the points *N′* and *N″* are the same as those points in Fig. 20. Newly designed are in Fig. 25 (as in Fig. 21), the longitudes from the intersection as angles and arcs and indeed with *a′* this longitude of the place *A′*, that in the smaller latitude, with *a″* this long. of the place *A″*, that in the higher latitude, if the places are on opposite sides of the Equator; but if they are on the same side, one of the two, *a′* or *a″*, that one of them which belongs to the transferred place, designs the supplement of the longitude required. The new construction embraces the two drawn radii *C2* & *C3*, the perpendiculars *Bn′* and *Bn″* as also the perpendiculars *n′9* & *n″11*, the length *n′10* & *n″12*, respectively equal to the last named perpendiculars *n′9* & *n″11*, and the lines *10B* and *12B*. The constructed angle *D10B* is equal to the spherical Course at the place *A′*, and *E12B* equal to that at the place *A″*.

Fig. 25.



55. If the spherical Course at the place *A′* is required, from o*B* drop the perpendicular *Bn′* upon *CD* and from its foot, *n′*, the perpendicular *n′9* upon *C2*; set off by the compasses *n′9* from *n′* upon *n′C* which determines the point *10*; draw *10B*, then the angle *D10B* is equal to the required Course.

If the spherical Course at the place *A″* is required, from o*B* drop the perpendicular *Bn″* upon *CE* and from its foot *n″* drop *n″11* perpendicularly upon *C3*; set off by the compasses *n″11* from *n″* upon *n″C* which determines the point *12*; draw *12B*, then the angle *E12B* is equal to the required Course.

Controls of the construction.

56. (1) Produce *n′9* both ways to the circumference, then half the chord (the distance from *9* to one of the points of intersection, measured on the producing of *sn′9*) must be equal to the hypotenuse *10B*.

Also the producing of *11n″* to the circumference measured from *11* to the intersection must be equal to the hypotenuse *12B*.

(2) In case the Inclination of the great circle to the equator is found by Construction, Fig. 21, the hypotenuses *62* and *10B* must be equal to each other; also the hypotenuse *83* equal to *12B*.

By Calculation,

by means of the place's latitude and longitude from the intersection of the great circle with the Equator. (Art. 46.)

[Solution of the right-angled spherical triangle. Appendix II, formula (5).]

Rule.

57. Add together the log. cotan. of that Long. (Table's Long.) of the given place and the log. sine of its latitude; the sum (rejecting ten) is the log. cotan. of the spherical Course at that place.

Ex. 1. To find by calculation the spherical Course at either of the extremities (the ship's place) of the great circle's arc between St. Helena and C. Horn. [Ex. 1, 1. Page 22, (X).]

When St. Helena is the ship's place.			When C. Horn is the ship's place.		
By means of St. Helena's			By means of C. Horn's		
Long. from the Inters.	10°33′	log. cot. 10·7298	Long. from the Inters.	72° 5′	log. cot. 9·5096
lat.	15°55	log. sin. 9·4381	lat.	55°59′	log. sin. 9·9185
the req. spher. Course	34°11′*)	log. cot. 10·1679	the req. spher. Course	75° 0′	log. cot. 9·4281
By means of Fig. 22,	S. 34°11′ W.		By means of Fig. 22,	N. 75° E.	

Ex. 2. To find by calculation the spherical Course at the ship's place being in the entrance of the English Channel, lat. 49° 6′ N., long. 6° 2′ W., and bound to New York. [Ex. 2, 1. Page 23, (XI).]

(Engl. Channel.) The adopted place's	Long. from the Inters.	68°13′	log. cot. 9·6017
	lat.	49° 6′	log. sin. 9·8784
	the required spherical Course	73°11′	log. cot. 9·4801
	By means of Fig. 23,	N. 73°11′ W.	

Ex. 3. To find by calculation the spherical Course at either of the extremities (the ship's place) of the great circle's arc between Panama and Port Jackson. [Ex. 3, 1. Page 24, (XII).]

When Panama is the ship's place.			When Port Jackson is the ship's place.		
By means of Panama's			By means of Port Jackson's		
Long. from the Inters.	12° 4′	log. cot. 10·6700	Long. from the Inters.	62°53′	log. cot. 6·7093
lat.	8°57′	log. sin. 9·1919	lat.	33°51′	log. sin. 9·7451
The req. spher. Course	53°57′	log. cot. 9·8619	The req. spher. Course	94° 5′	log. cot. 9·4552
By means of Fig. 24,	S. 53°57′ W.		By means of Fig. 24,	S. 74° 5′ E.	

Determination of the Spherical Distance.

58. To decide in the choice between the ship's tracks, the determination of the spherical distance between the ship's place and her place of destination is not required, after having proved that the great circle's arc is always the shortest line between any two places on the surface of the globe (Art. 19). Therefore we have not found necessary to indicate the distances on the tracks of the Scale of Great Circles **); wherefore the determination of the spherical distance between two given places by Inspection of the Blank Chart is here excluded.

59. Besides this, it is to be noticed, that the distance between a place which is one of the extremities of a determined great circle's arc and the following or the next preceding point determined by the *Tables* is to be found by the respective Mercator's Rule; also that the spherical distance between the two given places might be found by adding together those two distances between either of the places

*) This Course is more approximate than that of 34° 12′ determined by Napier's Analogy without considering the quantities of seconds exactly. Considering the seconds exactly, this Analogy gives 34° 11′ 18″ for that Course.

**) Such an indication is made on R. Russel's Diagram of Great Circles. London; published by Mrs. Janet Taylor, at her Nautical Academy and Navigation Warehouse, 104, Minories. 1852. — The Diagram with the accompanying General Chart substitutes in every respect a Terrestrial Globe the diameter of which is equal to 7 inches.

and the next point determined by the Tables and all the intermediate distances determined by them.

We shall however now give other methods of Solution of

PROBLEM III.

To find the spherical distance between the ship's place and her place of destination, *) *after having determined the great circle which passes through these places.* (*Chief Problem Art.* 31.)

BY CONSTRUCTION.

60. The solution is already executed by the straight line $A'A''$ Fig. 21 (Art. 39, 40 & 41) which is the chord of the required distance or of its supplement. The measure in degrees and minutes is to be taken by the scale of chords or the protractor applied in the solution. Reducing this measure into minutes, the distance is expressed in sea miles.

61. If the given places are on opposite sides of the Equator, the distances between either of them and the intersection of the great circle's track with the Equator may be asked for. For this case the solution is already executed by the second control of Fig. 21 [Art. 45, (2)] as also by the first control of Fig. 25 [Art. 56, (1)]. The producings there required cut directly on the circumference arcs which are, respectively, the spherical distances named. But we shall give the solution specially.

PROBLEM III. i.

To find by construction the spherical Distance between either of two given places which are on opposite sides **) *of the Equator and the point of Intersection of the Equat. with the great circle passing through those places.*

By means of the construction of the great circle's Inclination to the Equator. Fig. 21.

Produce the drawn perpendicular $N'5$, meeting the circumference; then the arc between either of the intersecting points and the point oB is the measure of the required distance of the place in the smaller latitude b'. (Proof Appendix II, Fig. 3, ii.)

Produce the drawn perpendicular $N''7$, meeting the circumference; then the arc between either of the intersecting points and the point oB is the measure of the required distance of the place in the higher latitude, b''. (Proof Appendix II, Fig. 3, ii.)

By means of the construction to find the Spherical Courses. Fig. 25.

Produce the drawn perpendicular $n'9$, meeting the circumference; then the arc between either of the intersecting points and the point *2* is the measure of the required distance of the place in the smaller latitude, b'. (Proof Appendix II, Fig. 4, ii.)

Produce the drawn perpendicular $n''11$, meeting the circumference; then the arc between either of the intersecting points and the point *3* is the measure of the required distance of the place in the higher latitude, b''. (Proof Appendix II, Fig. 4, ii.)

BY CALCULATION.

By means of the given places' latitudes and their longitudes from the Intersection of the Equator with the great circle passing through the given places. (Art. 46.)
(Solution of the right-angled spherical triangle. Appendix II, formula 3.)

Rule.

62. Add together the log. cosine of the latitude of either of the places and the log. cosine of the same place's Long. from the said Inter-

*) On the compound circle's track the first place of destination is not the port, the ship is bound for.

**) If the two given places are on the same side of the Equator, the solution gives the required Distance itself of the place not transferred, but the supplement of the required Distance of the place transferred. (Art. 30 & Page 46; Solution by Construction.)

section, the sum (rejecting ten) is the log. cosine of the spherical distance between the chosen place and the said Intersection; but when the applied longitude is *greater* than 90°, take for that distance the *supplement* of the angle found by the TABLE OF THE LOG. COSINES.

Find in the same manner the spherical distance between the other place and the said Intersection.

If the given places are on the same side of the Equator, subtract the shorter of the found distances from the greater, and reduce the difference into minutes; then this quantity of minutes expresses the required distance in sea miles.

But if the given places are on opposite sides, add together the two found distances, and reduce the sum into minutes; then this quantity of minutes expresses the required distance in sea miles.

Ex. 1. To find by calculation the Spher. Dist. between St. Helena and C. Horn. [Ex. 1, 1. Page 22, (X).]

By means of St. Helena's			By means of C. Horn's		
Long. from the Inters.	10°33′ W.	log. cos. 9·9926	Long. from the Inters.	72° 5′ W.	log. cos. 9·4880
lat.	15°55′	log. cos. 9·9839	lat.	55° 9′	log. cos. 9·7477
Spher. Dist. fr. the Int.	13° 1′	log. cos. 9·9756	Spher. Dist. fr. the Int.	80° 0′	log. cos. 9·2357

St. Helena's Spherical Distance from the Inters. 19 1

Spher. Dist. between St. Helena and C. Horn (diff.) 61° 4′ = 3664 miles.

Ex. 2. To find by calculation the Spherical Distance between the ship's place in the entrance of the English Channel, lat. 49° 6′ N., Long. 5° 2′ W. and New York. [Ex. 2, 1. Page 23, (XI).]

By means of the adopted place's (Engl. Channel)			By means of New York's		
Long. from the Inters.	68°23′ W.	log, cos. 9·5695	Long. from the Inters.	136°13′ W.	log. cos. 9·8585
lat.	49° 6′	log. cos. 9·8161	lat.	40°42′	log. cos. 9·8797
Spher. Dist. fr. the Int.	75°56′	log. cos. 9·3756	Spher. Dist. fr. the Int.	56°49′	log. cos. 9·7392
			Supplement	123 11	

The adopted place's Spher. Dist. from the Inters. 75 56

Spher. Dist. between the ship's adopted place and New York (diff.) 47°15′ = 7633 miles.

Ex. 3. To find by calculation the Spher Dist. between Panama and P. Jackson. [Ex. 1, 1. Page 24. (XII).]

By means of Panama's			By means of Port Jackson's		
Long. from the Inters.	12° 4′ E.	log. cos. 9·9903	Long. from the Inters.	117° 7′ W.	log. cos. 9·6588
lat.	8°57′	log. cos. 9·9947	lat.	33°51′	log. cos. 9·9193
Spher. Dist. fr. the Int.	14°58′	log. cos. 9·9650	Spher. Dist. fr. the Int.	67°45′	log. cos. 9·5781
			Supplement	112 15	

Panama's Spher. Dist. from the Inters. 14 58

Spherical Distance between Panama and Port Jackson (sum) 127°13′ = 7633 miles.

V.

DETERMINATION OF THE LIMIT OF THE POLAR TRACKS.

65. When two places are given (both of them neither on the Equator nor on one and the same meridian), and the great circle's arc between them is drawn on the surface of the globe itself, as also the respective part of the Rhumbline, a third curve can be drawn between the given places on the opposite side of that arc in a manner, that it is in all other respects located to the great circle's arc as the said Rhumbline. This opposite curve is called by us *„the Limit of the Polar Tracks.“* (Art. 85.)

If the adopted part of the Rhumb line is an arc of a Parallel, the limit of the polar tracks is an arc of a small circle equal to that parallel circle, and if the Rhumb line is a Spiral, the limit of the polar tracks

is also a spiral, but in both cases these opposite curves are not Rhumb lines, they do not cross the *meridians* at the same inclination. Instead of this, they have the same inclination to the great circles of a system having their common intersections at the poles of the great circle which has an inclination to the Equator equal to twice that inclination of the great circle passing through the two given places. These poles lie in the meridian of the vertex of the great circle which passes through the two given places, the distance of each of them from one of the Poles of the Earth is equal to the supplement of twice the inclination of the great circle (determined by the two given places) to the Equator.

66. Fig. 26 represents a Hemisphere of the Globe. It is drawn in the same mode of projection as the Map of the World in two Hemispheres. *) It shews as a straight line, $BA'A''\ldots$, a semicircle of that great circle of which the Inclination to the Equator is equal to 50 degrees. The Pole of the Earth is designed with P. The equator line is represented by the arc $BR'R''\ldots$, graduated as scale of the longitudes reckoned from the Intersection of the said great circle with the Equator. The meridian of the Vertex of that circle is represented by the straight line $Pp\ldots$, and graduated to indicate the latitudes. The other meridians are drawn through each thirtieth degree of longitude (besides these, parts of some others) and the parallels to each thirtieth degree of latitude in the adopted mode of projection.

Let A' & A'' be two places **) on the great circle of which the inclination to the Equator is equal to the angle $B = 50°$, and $A'SA''\ldots$ the Rhumb line (Art. 18) passing through them both and winding towards P, the Pole of the Earth; then $R'A'$ & $R''A''$ are, respectively, the latitudes of A' & A'', BR' & BR'' the longitudes of them reckoned from B; $BA'R'$, $A'c'S'$, $A'c''S''$, &c. the spherical Courses at the places A', c', c'', &c.; $c'S'$, $c''S''$, $c'''S'''$, &c. the differences between the latitudes of c', c'', c''', &c., points on the great circle's arc, and S', S'', S''', &c., points of the Rhumbline, respectively, in the same meridians.

Again draw the great circle $Br'\,80\,r''\ldots$ at an inclination to the Equator $= 2 \times B = 2 \times 50° = 100°$; mark p, the pole of that circle, on the common Merid. of Vertexes of the first adopted and the last drawn great circle; draw the quadrants from p through A', c', c'', &c., meeting the circle $Br'r''$; lastly draw a curve which passes through A' & A'' and crosses those quadrants at the same inclination which

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*) This mode of projection is mathematically speaking called „*Stereographical Projection*"; it is a „*linear perspective*" imagining the Earth transparent. The plane of projection is always a great circle and the position of the eye is in one of the poles of it; the represented hemisphere is, in relation to the eye, on the opposite side of the plane of projection.

To represent the *Eastern* or *Western Hemisphere*, the plane of projection is that of one and the same meridian adopted and the position of the eye is on the Equator 90° distant from the intersection of it with that meridian. The meridian in which the eye lies and the Equator are represented on the map by straight lines; the other meridians by excentric arcs which have their centres on the equator line and its prolongations, and the Parallels by such arcs the centres of which are on that straight lined meridian and its prolongations.

To represent the Northern or Southern Hemisphere, the plane of projection is that of the Equator and the eye lies for each of them in the opposite Pole of the Earth. The Parallels are represented by concentric circles the centre of which is that of the map, and the meridians by straight lines which cross each other in that centre.

In Fig. 26 the plane of projection is that of the great circle the poles of which are the Vertexes of the great circle of which the inclination to the Equator is equal to 50°. The position of the eye is one of those Vertexes. The meridian of Vertex is represented by a straight line, all the other ones by excentric arcs of which the centres lie on a straight line which is perpendicular to the projection of the meridian of Vertex and situated beyond the projection of the Equator with respect to the Pole of the Earth. The Equator and the Parallels are represented by excentric arcs (respectively by such circles) the centres of which lie on the projection of the meridian of Vertex and its prolongation.

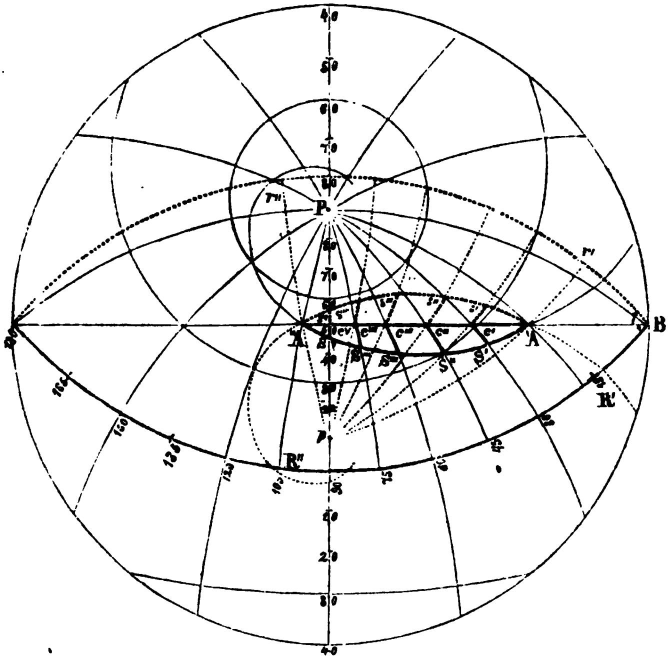
The position of the centre and the length of the radius of every one of those arcs (circles) are to be found by calculation.

The latitudes as also the longitudes are on the map in the middle of the Hemisphère smaller than towards its border.

**) A' and A'' are here adopted in unequal latitudes. If they lay in equal latitudes, the Rhumbline would be an arc of a Parallel, but in this case the following proof remains unaltered; only the application would be more simple, because the Mer. of Vertex bisects such a Rhumbline.

the Rhumbline passing through A' & A'' has to all the meridians as it crosses them in succession: then we have by the equality of the respective spherical triangles

Fig. 26.



(1) the arc $A'r' = A'R'$ & $A''r'' = A''R''$;
(2) the arc $c's' = c'S'$, $c''s'' = c''S''$, $c'''s''' = c'''S'''$, &c.;
(3) the angle $BA'r' = BA'R'$ & $BA''r'' = BA''R''$;
(4) the angle $A'c's' = A'c'S'$, $A'c''s'' = A'c''S''$, $A'c'''s''' = A'c'''S'''$, &c.;
(5) the angle $Pc's' = (180° - 2 . A'c'S')$, $Pc''s'' = (180° - 2 . A'c''S'')$, &c.

67. Therefore, from every point on the great circle's arc between two given places, a point of the Limit of the Polar Tracks is determined by *distance*, (2), and *direction*, (5).

This *distance* is always equal to the difference between the latitude of the point on the great circle and the latitude on the Rhumbline in the meridian of that great circle's point.

This *direction* is determined by a Compass Bearing (Terrestrial Azimuth) equal to the supplement of twice the spherical Course at that great circle's point, and is to be reckoned from the N., if the given place in the term of the track lies on the Northern Hemisphere, to be reckoned from the S., if that place lies on the Southern Hemisphere; besides this, westwards, if the respective great circle's point lies westwards from the Vertex of that great circle, but eastwards if it lies eastwards from that Vertex.

If the given places are on the same side of the Equator, all those bearings are northern or southern like the Hemisphere on which the track lies. But if the given places are on opposite sides of the Equator, those bearings change with respect to N. and S. at the intersection of the great circle's arc with the Rhumbline (not at its intersection with the Equator).

To find these two elements, having determined the respective points on the great circle's arc as also the Rhumb Course between the two given places; only known rules are to be applied, but we shall specially notice the solution.

Problem IV.

To find the distance of the point on the Limit of the Polar Tracks from any point determined on the great circle's arc, and the bearing of this distance.

By Calculation.

68. (1) For the Distance. Find the Rhumb Course and D. Long. between the given place in the smaller latitude and the adopted point of the determined ones on the great circle's arc.

To the log. of that D. Long. add the log. cot. of the Rh. Course; the sum (rejecting ten) is the log. of the respective Mer. D. Lat.

To that Mer. D. Lat. add the meridional parts of the latitude of the given place in smaller latitude; the sum is equal to the meridional parts of the latitude on the Rhumb line in the meridian of the adopted point on the great circle's arc.

The lat. corresponding to those meridional parts subtract from the lat. of the great circle's point; the difference is equal to the required distance.

Ex. 2. Being in the entrance of the English Channel, Lat. 49° 6′ N., Long. 6° 2′ W., and bound to New York, Lat. 40° 42′ N., Long. 74° 2′ W. — The Rhumb Course = S. 80° 5′ W. —

The spher. Course = $6\frac{7}{8}$ Pts. takes place on the great circle's arc, [Pag. 30. (XIV); Table (1).]

	in Lat. 50°3′ and Long. 11°29′.		in Lat. 50°3′ and Long. 44°9′.	
N. York's long.	74° 2′		74° 2′	
Circ. points long.	11 29		44 9	
D. Long.	62°33′ = 3753′	log. 3·5743	29°53′ = 1793′	log. 3 2533
Rhumb Course .	80° 5′	log. cot. 9·2426		9·2426
log. Mer. D. Lat.		(sum reject. 10) 2·8169		(sum reject. 10) 2·4959
Mer. D. Lat.		656		313
N. York's lat. 40°42′ merid. parts		2678		2678
Rh. place's lat. merid. parts		(sum) 3334		(sum) 2992
Rh. place's true lat.		48°28′		44°33′
Circ. place's lat.		50 3		50 3
required distance		(diff.) 1°35′		(diff.) 5°30′

(2) For the Bearing. Take twice the respective spherical Course and subtract the product from 16 Pts.; the difference is the measure of the required bearing (Terrestrial Azimuth).

In the preceding Example the *distance* is calculated with respect to the points on the gr. circle at which the spherical Course is equal to $6\frac{7}{8}$ Pts. Thus, we have, $6\frac{7}{8}$ Pts. $\times\ 2 = 13\frac{3}{4}$ Pts., and $13\frac{3}{4}$ Pts. to subtract from 16 Pts. = $2\frac{1}{4}$ Pts., equal to the required bearing. Looking at the sketch Fig. 23 (the Blank Chart or the Chart itself, if the great circle's arc be determined on either of them) the bearing at the great circle's point in longitude 11°29′ is N. $2\frac{1}{4}$ p. E., that at the great cirlce's point in longitude 44°9′ is N. $2\frac{1}{4}$ p. W.

By Construction.

69. (1) For the Distance. After having marked on the Chart the points of the great circle's arc which are determined by the accompanying Tables [Ex. 2. Page 30 (XIV), either Extract (2) or (3)], or on the Blank Chart those points, determined by the spherical Courses indicated on the two curves between which the great circle's arc lies, draw the Rhumb line between the given places, and the parts of the meridians from those great circle's points to that Rhumb line; then

every one of the required distances is to be taken as D. Lat. between the extremities of each of those parts of the meridians, in the known manner, from the scale of latitudes (the graduated meridian).

(2) For the Bearing. Its determination is completely explained, Art. 97 & 68 (2). To trace it on the Chart is easely done with the parallel ruler, because it is expressed at least in fourths of the Point, employing the accompanying Great Circle Tables.

70. But for the practical application of points on the Limit of the Polar Tracks their determination by means of distance and bearing, with reference to the determined place, is not directly sufficient. In practice, the position of every one of those points must be determined by its Lat. and Long. or by its trace on the Chart.

71. The values of such a Lat. and Long. may be found to a sufficient degree of approximation, by determining them after Mercators Rule; which leads to a solution like that of the known Problem,

„*One Latitude, Course, and Distance given, to find the Difference of Latitude and Difference of Longitude*"

in which problem, there is to be substituted, in our case, for the *given Lat.*, the Lat. of the determined place on the great circle's arc; for the *Course*, the bearing; for the *given Dist.*, the distance between the point on the great circle and the required point on the Limit of the Polar Tracks; and for *D. Lat.* and *D. Long.*, D. Lat. and D. Long. between the just named two points.

Ex. 2. Being in the entrance of the English Channel, Lat. 49° 6′ N., Long. 6° 2′ W., and bound to New York, Lat. 40° 42′ N., Long. 74° 2′ W.

The spherical Course = $6\frac{7}{8}$ Pts. takes place on the great circle's arc [pag. 30, (XIV); Table (1)]

in Lat. 50°3′ and Long. 11°29′.	in Lat. 50°3′ and Long. 44°9′.
The Dist. = 1°35′; the Bear. = N. $2\frac{1}{4}$ Pts. E. (Art. 68.)	The Dist. = 5°30′; the Bear. = N. $2\frac{1}{4}$ Pts. W. (Art. 68.)
To find D. Lat.	
Dist. 1°35′ = 95′ log. 1·9777	5°30′ = 330′ log. 3·5185
Bearing $2\frac{1}{4}$ Pts. log. cos. 9·9562	9·9562
D. Lat. 86′ log. (sum reject. 10) 1·9339	298′ log. (sum reject. 10) 2 4747
To find the Lat.	
Lat. given 50° 3′ Mer. parts 3479	50° 3′ Mer. parts 3479
D. Lat. . . 1°26′	4°58′
Lat. req. (sum) 51°29′ Mer. parts 3615	(sum) 55° 1′ Mer. parts 3970
Mer. D. Lat. (diff.) 136	(diff.) 491
To find D. Long.	
Mer. D. Lat. 136 log. 2·1335	491 log. 2·6911
Bearing $2\frac{1}{4}$ Pts. log. tan. 9·6748	9·6748
D. Long. 64′ log. (sum reject. 10) 1·8083	232′ log. (sum reject. 10) 2·3659
To find the Long.	
Long. given . 11°29′ W.	44° 9′ W.
D. Long. . . . 1° 4′ E.	3°52′ W.
Long. requir. (sum) 10°25′ W.	(sum) 48° 1′ W.

72. *To trace the required point on the Chart* is not a proper problem, if the Lat. and Long. of the required point are already found by calculation Art. 71.

The solution is however shortened by excluding the calculation of D. Long., tracing on the Chart the Bearing and the Parallel of the required point, after having found the Lat. of this point by calculation Art. 71.

The direction which is to be laid down upon the Chart by the Bearing meets the Parallel at an angle not too acute, to afford accurately the re-

quired point of the Limit of the Polar Tracks, if the spherical Course on the great circle's arc lies between $5^1/_2$ and 8 Pts., therefore always over the top of the great circle, as also if that Course lies between *zero* and $2^1/_2$ Pts. However having calculated the longitude, the drawn Meridian meeting the direction laid down by the Bearing affords a correct point of that Limit in all cases, where the Parallel does not.

In general the problem might be solved by the more or less approximate modes of solution of the known problem,

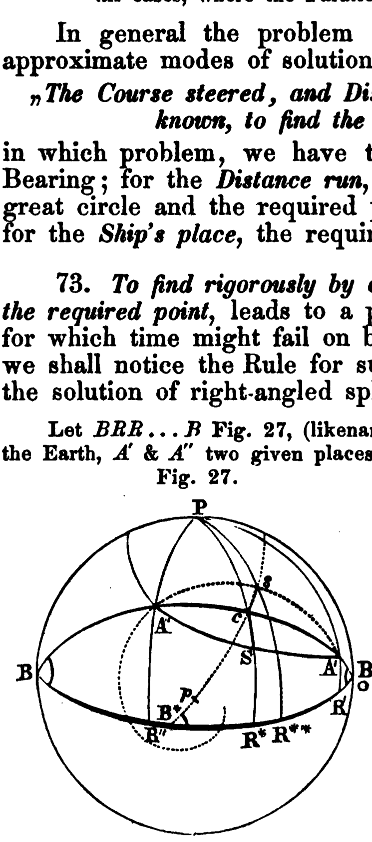
„*The Course steered, and Distance run from any given Place, being known, to find the Ship's Place on the Chart*“;

in which problem, we have to substitute for the *Course steered*, the Bearing; for the *Distance run*, the Distance between the point on the great circle and the required point on the Limit of the Polar Tracks; for the *Ship's place*, the required point of that Limit.

73. *To find rigorously by calculation the Latitude and Longitude of the required point*, leads to a process which might be too tedious and for which time might fail on board. Yet to complete our investigation, we shall notice the Rule for such a calculation and reason it out upon the solution of right-angled spherical triangles.

Let $BRR\ldots B$ Fig. 27, (likenamed in Fig. 26) be the Equator, P the Pole of the Earth, A' & A'' two given places, $BA'A''B$ the great circle passing through them,

Fig. 27.



$A'SA''\ldots$ the Rhumb line (here a Spiral), $A'sA''$ the respective part of the spiral on the opposite side of the great circle's arc, p the pole towards which this spiral winds, and c any point on the great circle's arc $A'A''$.

Draw the arc of the great circle which passes through c and p, meeting the Equator in B^* and the opposite spiral in s; and draw the meridian PcR^*; then,

in the right-angled spherical triangle B^*R^*c,

the leg R^*c is the Lat. of the point c,

the angle $B^*cR^* = Pcs =$ the supplem. of twice the spher. Course at the point c, [Art. 66 (5)]

the leg $R^*B^* =$ D. Long. between R^* and c, and opposite to the angle B^*cR^*,

$\therefore$ tan. $R^*B^* =$ tan. $B^*cR^* \times$ sin. R^*c; [App. II (5)]

the angle B^* is opposite to the leg R^*c,

$\therefore$ cos. $B^* =$ sin. $B^*cR^* \times$ cos. R^*c; [App. II (8)]

B^*c is equal to the Dist. from B^* to c, and the hypotenuse,

$\therefore$ tan. $B^*c =$ sec. $B^*cR^* \times$ tan. R^*c. [App. II (7)]

Drawing the meridian PsR^{**}; then

in the right-angled spherical triangle $B^*R^{**}s$,

B^*s is the hypotenuse and $= B^*c + cs = B^*c + cS$, [Art. 66 (2)]

but $R^{**}s$ the Lat. of the required point, and $R^{**}B^*$ the D. Long. betw. that point, s, and B^*,

$\therefore$ sin. $R^{**}s =$ sin. $B^* \times$ sin. $(B^*c + cS)$; [App. II (1)]

$\therefore$ tan. $R^{**}B^* =$ cos. $B^* \times$ tan. $(B^*c + cS)$. [App. II (6)]

Finally, we have

$R^{**}R^* =$ D. Long. between s and c,

equal to the difference between $R^{**}B^*$ and R^*B^*, the two above determined Ds. Long.

This Proof leads to the

Rule.

74. (1) Find by calculation the distance of the required point *s*, from *c*, the point on the great circle's arc, to which *s* is referred, and the Bearing of that distance. Art. 68.

(2) To the log. tan. of the Bearing add the log. sine of the Lat. of the point *c* on the great circle's arc, the sum (rejecting ten) is the log. tan. of the D. Long. B^*R^* (between *c* and B^*, one of the two Intersections of the Equator with the great circle fixed by *c* and the Bearing).

(3) To the log. sine of the Bearing add the log. cos. of the Lat. of that point *c*, the sum (rejecting ten) is the log. cos of the angle B^*, the Inclination of the great circle B^*pcs (fixed by the point *c* and the Bearing) to the Equator.

(4) To the log. sec. of the Bearing add the log. tan. of the Lat. of the point *c*, the sum (rejecting ten) is the log. tan. of the spherical distance between *c* and the point B^*.

(5) To this distance B^*c, found by (4), add the distance $cs = cS$, found by (1), the sum is the spherical distance B^*s, that of the required point, *s*, from the Intersection B^*.

(6) To the log. sin. of the Inclination B^*, found by (3), add the log. sin. of the distance B^*s, found by (5), the sum is the log. sin. of the *Lat. of the required point, s.*

(7) To the log. cos. of the Inclination B^*, found by (3), add the log. tan. of the distance B^*s, found by (5), the sum (reject. ten) is the log. tan. of the D. Long. B^*R^{**} (between the required point, *s*, and the point B^*).

(8) From the D. Long. found by (7) subtract the D. Long. found by (2), the difference is the *D. Long.* R^*R^{**} (between the required point, *s*, and *c*, the point on the great circle's arc to which *s* is referred).

(9) To the known Long. of the point *c* *add algebraically* the *D. Long.* found by (8), the sum is the *Long. of the required point s.*

Ex. 2. Being in the entrance of the English Channel, Lat. 49° 6′ N., Long. 6° 2′ W., and bound to New York, Lat. 40° 42′ N., Long. 74° 2′ W.

The spherical Course = 6 $\frac{7}{8}$ Pts. takes place on the great circle's arc. [Page 30, (XIV), Table (1).]

Lat. 50°3′ N. and Long. 11°29′ W.		Lat. 50°3′ N. and Long. 44°9′ W.	
1. (Art. 68.) The Dist. cs = 1°35′; the Bear. = N. 2¼ Pts. E.		The Dist. cs = 5°30′; the Bear. = N. 2¼ Pts. W.	
1. Bearing 2¼ Pts.	log. tan. 9·6748		
given Lat. 50°3′	log. sin. 9·8846		
2. D. Long. B^*R^* = 19°56′	log. tan. (sum rej. 10) 9·5594	= 19°56′	
1. Bearing 2¼ Pts.	log. sin. 9·6310		
given Lat. 50°3′	log. cos. 9·8076		
3. Inclin. B^* = 74°4′	log. cos. (sum reject. 10) 9·4386	= 74° 4′	
1. Bearing 2¼ Pts.	log. sec. 10·0438		
given Lat. 50°3′	log. tan. 10·0770		
4. Dist. B^*c = 52°52′	log. tan. (sum rej. 10) 10·1208	= 52°52′	
1. Dist. cs = 1 35		= 5 30	
5. Dist. B^*s (sum) 54°27′		= (sum) 58°22′	
3. Inclin. B^* = 74° 4′	log. sin. 9·9830		log. sin. 9·9830
5. Dist. B^*s = 54°27′	log. sin. 9·9104	= 58°22′	log. sin. 9·9301
6. req. Lat. = 51°29′N.	log. sin. (sum reject. 10) 9·8934	= 54°57′N.	log. sin. (sum rej. 10) 9·9131
3. Inclin. B^* = 74° 4′	log. cos. 9·4386		log. cos. 9·4386
5. Dist. B^*s = 54°27′	log. tan. 10·1459	= 58°22′	log. tan. 10·2104
7. D. Long. B^*R^{**} = 21° 1′	log. tan. (sum rej. 10) 9·5845	= 24° 1′	log. tan. (sum rej. 10) 9·6490
2. D. Lg. B^*R^* = 19°56′		= 19°56′	
8. D. Lg. R^*R^{**} = (diff.) 1° 5′ E.		= (diff.) 4° 5′ W.	
Long. of c = 11 29 W.		= 44 9 W.	
9. req. L. (of s) = (sum) 10°24′ W.		= (sum) 48°14′ W.	

75. To illustrate, how much the Courses and the Latitudes would differ on the Rhumb line, the great circle's arc and the limit of the polar tracks between the places adopted in Ex. 2., we now place together tables containing these lines, without respect to their being navigable or not, which will be considered in „The Sailing“.

Besides this, we have calculated that limit of the polar tracks in the two given manners, to shew the difference of the values thereby.

Curves

between the place, lat. 49°6′ N., long. 6°2′ W., and the place, lat. 40°42′ N., long. 74°2′ W. on the Surface of the Globe.

No. 4.

The Rhumb line referred to the gr. circle's arc No. 3. Calculat. of the Lats. Art. 68.

Course	Lat. °	Lat. ′	Long. fr. Greenw. °	Long. fr. Greenw. ′	Dist. Miles
S. 80° 5′ W.	49	6	6	2	70
	48	54	7	48	151
	48	28	11	29	145
	48	3	15	9	151
	47	37	18	47	151
	47	11	22	24	151
	46	45	26	1	151
	46	19	29	37	151
	45	53	33	14	151
	45	27	36	51	157
	45	0	40	29	157
	44	33	44	9	162
	44	5	47	50	163
	43	37	51	33	168
	43	8	55	19	174
	42	38	59	9	174
	42	8	63	3	180
	41	37	67	3	186
	41	5	71	10	134
	40	42	74	2	
Dist. on the Rhumbline				(Sum)	2927

No. 3.

The Great circle's Arc. The Stages are Chords on the Chart. Page 30.

Course	Lat. °	Lat. ′	Long. fr. Greenw. °	Long. fr. Greenw. ′	Dist. Miles
N 73° 12′ W	49	6	6	2	73
N 75° 51′ W	49	27	7	48	147
N 78° 43′ W	50	3	11	29	143
N 81° 47′ W	50	31	15	9	139
N 84° 33′ W	50	51	18	47	138
N 87° 4′ W	51	4	22	24	136
West	51	11	26	1	136
S 87° 4′ W	51	11	29	37	136
S 84° 33′ W	51	4	33	14	138
S 81° 47′ W	50	51	36	51	139
S 78° 43′ W	50	31	40	29	143
S 75° 51′ W	50	3	44	9	147
S 72° 58′ W	49	27	47	50	153
S 70° 36′ W	48	42	51	33	160
S 67° 23′ W	47	49	55	19	169
S 64° 37′ W	46	44	59	9	179
S 61° 56′ W	45	27	63	3	194
S 59° 6′ W	43	56	67	3	210
S 56° 18′ W	42	8	71	10	155
	40	42	74	2	
Dist. on the great circle's arc				(Sum)	2835

No. 5.

The Limit of the Polar Tracks referred to the great circle's arc No. 3. Calc. only by Merc. Rules. Art. 68 & 71.

Course	Lat. °	Lat. ′	Long. fr. Greenw. °	Long. fr. Greenw. ′	Dist. Miles
N 46° 39′ W	49	6	6	2	71
N 50° 58′ W	49	55	7	22	149
N 57° 4′ W	51	29	10	25	149
N 62° 22′ W	52	50	13	49	149
N 68° 14′ W	53	59	17	30	148
N 73° 19′ W	54	54	21	27	150
N 79° 59′ W	55	37	25	39	149
N 86° 35′ W	56	3	30	1	151
S 87° 22′ W	56	12	34	31	152
S 81° 47′ W	56	5	39	3	154
S 74° 28′ W	55	43	43	35	157
S 68° 44′ W	55	1	48	1	160
S 62° 13′ W	54	3	52	18	163
S 57° 32′ W	52	47	56	20	168
S 51° 22′ W	51	17	60	10	173
S 46° 17′ W	49	29	63	42	178
S 40° 42′ W	47	26	66	56	183
S 36° 50′ W	45	7	69	49	190
S 32° 8′ W	42	35	72	27	133
	40	42	74	2	
Dist. on the Limit of the Polar Tracks				(Sum)	2927

No. (5)

The Limit of the Polar Tracks referred to the great circle's arc No. 3. Lats. and Longs. are calculated by Rules of spher. Trigonometry. (Art. 74.)

Course	Lat. °	Lat. ′	Long. fr. Greenw. °	Long. fr. Greenw. ′	Dist. Miles
N 46° 18′ W	49	6	6	2	71
N 50° 58′ W	49	55	7	21	149
N 56° 49′ W	51	29	10	24	148
N 62° 28′ W	52	50	13	46	149
N 68° 14′ W	53	59	17	28	148
N 73° 23′ W	54	54	21	25	150
N 80° 5′ W	55	37	25	38	151
N 86° 59′ W	56	3	30	3	152
S 87° 24′ W	56	11	34	35	154
S 81° 11′ W	56	4	39	11	
				1. Sum	1272

Continuation of No. (5).

Course	Lat. °	Lat. ′	Long. fr. Greenw. °	Long. fr. Greenw. ′	Dist. Miles
					157
S 74° 11′ W	55	40	43	47	158
S 68° 31′ W	54	57	48	14	161
S 61° 52′ W	53	58	52	32	163
S 57° 15′ W	52	41	56	33	166
S 51° 9′ W	51	11	60	20	171
S 46° 16′ W	49	24	63	48	177
S 40° 58′ W	47	22	67	0	181
S 36° 31′ W	45	5	69	52	188
S 32° 22′ W	42	34	72	27	133
	40	42	74	2	
				2. Sum	1655
				1. Sum	1272
Dist. on the Limit of the Polar Tracks				Sum	29 7

THE SAILING.

I.

COMPARISON OF DIFFERENT KINDS OF SHIP'S TRACKS.

The Great Circle's Track is the Pattern Track.

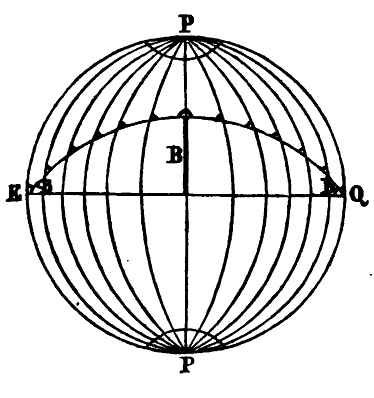
76. The ship which steers along a Rhumb line keeps the Course constant; but, if that Rhumb line is neither a portion of the Equator nor of any Meridian, she changes perpetually the Direction of her head with respect to her place of destination, being successively in different Vertical Planes of which only the first passes through the ship's place at the beginning of the track, only the last passes through her place of destination. (Art. 18.)

The ship which steers along a Great Circle's arc keeps the Direction of her head constant with respect to her place of destination, running continually in one and the same Vertical Plane, in that which passes through the extremities of that arc (Art. 17); but if the great circle's arc is neither a portion of the Equator nor of any Meridian, she changes perpetually her Course which does not remain the same between two places even a single fathom distant from each other. (Art. 77.)

The track on the great circle's arc between the ship's place at the beginning of the track and her place of destination is the shortest of all between those places (Art. 19), wherefore it must be the ***Pattern Track*** for all others.

Arcs of equal length on the same great circle.

77. Every Great Circle on the surface of the Earth is bisected by the Equator into a Northern and a Southern Semicircle [Art. 2, (1) & Art. 22]. If the circle is not a meridian, on every one of those semicircles, the Courses as well as the latitudes increase continually in the way from each of the circle's Intersections with the Equator towards the Vertex, and decrease continuing the way from the Vertex towards the second Intersection. (Art. 13.)



(1) The *Course* at every one of those Intersections is equal to the complement of the circle's inclination to the Equator; it is the *smallest* which can take place on the great circle. The Course at the Vertex of the circle is always a right angle; it is the *greatest* which can take place on the circle, reckoning the Course in the usual manner.

If from the Vertex of a great circle, the places at which the Course has decreased by an adopted invariable part are determined, any quadrant of that circle will be divided into the same number of parts. Of the Dists. as well as of the Ds. Long. between the extremities of such intervals, those on the Vertex are the shortest and both, Dist. and D. Long., increase towards the intersection of the circle with the Equator.

(In the accompanying TABLES the adopted changing of the Course corresponds to $^1/_8$ Point, on the Blank Chart to $^1/_4$ Point.)

(2) The *Latitude* of each of the two Vertexes of the great circle is equal to the Inclination of that circle to the Equator. It is the greatest of the Lats. of any places on the circle. The Lats. of the other places lie between *zero* and the Lat. of Vertex. On every side of the Equator, always two of these latitudes are equal to each other and at the same time equidistant from the points of intersection of the circle with the Equator.

(3) Equal Courses on the great circle correspond to equal latitudes of which two are N., and two S.. Reckoning the longitudes from the same Intersection of the circle with the Equator, the longitudes of those two places are supplements of each other. Let *that longitude* of a place on the circle be „*n* degrees W., (E.)", then the equal latitudes correspond, respectively, to the *longitudes*, „(180—*n*) degrees W., (E.)"; „*n* degrees E., (W.)"; and „(180—*n*) degrees E., (W.)".

From the preceding it follows:

78. (1) The *Changing of the Course* on arcs of equal length of one and the same great circle is greatest, if the middle of the arc coincides with one of the Vertexes of the circle; it is least, if that middle coincides with one of the Intersections of the circle with the Equator. In all other positions of the arc on the circle, the Changing of the Course on the arc is as much greater as the position of its middle is nearer to either Vertex of the circle.

(2) With respect to the places which are the extremities of those arcs, the Changing of the Course on equal arcs is always as much greater as the D. Long. between the two places is greater; or,

if the two places are on the same side of the Equator, that Changing is as much greater as D. Lat. is smaller;

but, if they are on opposite sides, that Changing is as much greater as the lower of the two given Lats. is smaller.

Arcs of equal length on different great circles.

79. The great circles differ from each other by their Inclinations to the Equator (Art. 22). The complement to that inclination is the smallest Course on the circle (Art. 77. 1). The difference between a right angle, as Course at each of the Vertexes, and the smallest Course

on the great circle is therefore as much greater as that inclination of the circle; for, the complement of an acute angle is less, if the angle itself is greater.

But the greater this difference is, so much the greater is on the circle the number of intervals *) in which the Course changes by an adopted invariable part.

Therefore we have: —

> The *Changing of the Courses* on arcs of equal length on different great circles is as much greater as the inclination of the circle to the Equator is greater, supposing those arcs in alike positions, with respect to the respective Vertexes. Art. 78, 1.

Comparison of the route on the great circle's arc with that on the Rhumb line between two given places.

80. Referring the qualities of the great circle's arc explained above to the known qualities of the Rhumb line, we have in general: —

(1) The greater the *Changing of the Courses* on the great circle's arc is, so much the more circuitous is the route on the Rhumb line between the same two places.

(2) The track on the great circle's arc and that on the Rhumb line *coincide*, if their common extremities are places either both on the Equator or both on any meridian. They *coincide nearly* in low latitudes; in all latitudes, if the D. Long. is very small; and always, if the Dist. is short. But although the distances on the two tracks may not yet differ from each other in practice, the differences between their Courses may require to be already considered, with respect to the Lats. and Longs. which will be reached.

On the other hand, the tracks differ *most widely* if the Rhumb track is a portion of a parallel; still *widely*, if the great circle's arc passes yet through the Vertex of the great circle, if the distance between the given places is considerable, and the latitudes of the given places are not too low.

If the two tracks cross each other, their respective portions which have their extremities on the same side of the Equator [Art. (3)] may *coincide nearly*, while the difference between their portions which have their common extremities on the opposite sides of the Equator is great enough to be considered.

(3) If the given places are on the same side of the Equator the route on the great circle's arc and that on the Rhumb line between them differ as much more from each other

as the D. Long. is greater,
as the D. Lat. is smaller,
as the lower of the given Lats. is higher.

If the given places are on opposite sides of the Equator, only the first of these three conditions is always to be applied, besides this, those routes differ as much more from each other

as the lower of the given latitudes is smaller.

81. For the sake of clearness in the determination of the mutual position of the two routes, we shall distinguish different cases.

*) The accompanying TABLES as well as the curves on the BLANK CHART illustrate serieses of such intervals on different great circles. — Description of the Tables, page 9; that of the Scale of Great Circles on the Blank Chart, page 41.

(1) If the two given places are on the same side of the Equator, it is evident, that the portion of the Rhumb line between them meets the great circle's arc, only in those two places.

(On the Chart the Rhumb line is, in this case, the chord of the curve which represents the great circle's arc. The small Charts adjoined to the TITLE shew several such routes; the respective great circle's arcs are represented by curves not dotted, and in the two cases in which more than one curve not dotted are drawn, that which is the most distant from the respective Rhumb line represents the great circle's arc.)

The Rhumb line lies towards the Equator with respect to the great circle's arc; and

when a ship steers along the great circle's arc she is always in a higher latitude than she would be in the same longitude (on the same meridian) on the Rhumb line.

(2) If the two given places are on opposite sides of the Equator, and their latitudes equal to each other, it is evident, that the great circle's arc and the Rhumb line between them cross and bisect each other in the intersection of the great circle with the Equator, and

the relations between the two routes are for either of the stages from the cutting point of the routes to each of the given places the same which are given in (1).

(3) Now supposing one of the given places fixed, but the D. Long. somewhat greater than before and the second given place remaining on the same great circle, the cutting point of the two routes falls nearer to the place fixed and the other place will be in higher latitude than the first named.

The routes are divided into two stages of which the one, that from the place in lower latitude to the cutting points of the two routes,

gives the relation of the case (1)

because the two extremities are on the same side of the Equator.

The other stage from the place in higher latitude to the cutting point of the two routes

shall be explained in the following, (4).

Such a case (3) is illustrated on the TITLE CHART No. 1 by the tracks between New York and the Cape of good Hope. The latter place is that in lower latitude, wherefore the cutting point of the great circle's arc (represented by the curve not dotted) with the Rhumb line lies on the S. Hemisphere, as also the portions of those tracks between two points on the same side of the Equator.

(4) Increasing D. Long. more and more, the cutting point of the two routes falls nearer and nearer to the place fixed, until that point coincides with this place. If this takes place, it is evident, that the great circle's arc and the Rhumb line between them do not cross each other.

Fig. 28.

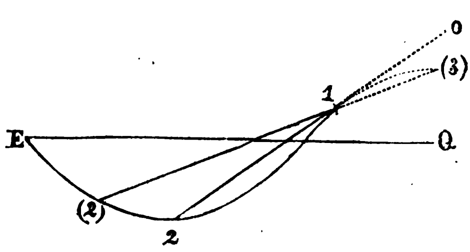


Fig. 28 shews such tracks in Mercator's Projection. The given place in lower latitude supposed as fixed in the above investigation is marked with *1* that in higher latitude with *2*. The straight line *21* represents the Rhumb track of which the prolongation *10* does not meet the great circle in a third point; it touches the projection of the respective great circle in the point *1*.

When in this case the ship steers along the great circle's arc she is, being between the given place in *higher* latitude and the meridian of the cutting point of the Rhumb line with the Equator, always in higher latitude than she would be in the same longitude, if steering on the Rhumb line.

Steering on the portion of the great circle between the given place in *lower* latitude and the meridian of the intersection of the great circle with the Equator, the ship is always in lower latitude than she would be in the same longitude, if steering on the Rhumb line.

Finally, when the ship steers along the portion of the great circle's arc between the meridian of the cutting point of the Rhumb line with the Equator and the meridian of the intersection of the great circle with the Equator, she is always in latitudes alike named with the higher of the two given latitudes; whilst steering on the portion of the Rhumb line between those two meridians she would be in latitudes alike named with the lower of the two given latitudes; therefore on the different routes in contrary latitudes.

(5) Continuing the supposed increasing of D. Long.; the Rhumb line will again meet the great circle in a third point, but this point lies in the prolongations of both the tracks, and the routes on them between the two given places remain without intersection.

Wherefore we have the relation given in (4).

Fig. 28 shews the great circle's arc and the Rhumb line for this case by the tracks drawn between the places *1* and *(2)*; the prolongation of the Rhumb line meets that of the great circle in the point *(3)*.

The great circle's arc between Panama and Port Jackson (treated in Ex. 3) gives such a track which does not cross the Rhumb track between the given places.

82. When the ship steers along a great circle's arc she reaches latitudes higher than those on the Rhumb line between the extremities of that arc, only if that great circle's arc passes through the Vertex of the respective circle, and in this case, the greatest latitude which she may reach is the latitude of that Vertex. But this determination as well as the above of the Ds. Lat. between the points on the great circle's arc and the Rhumb line, respectively, in the same longitudes (Art. 81), does not suffice for the choice between these tracks with respect to the known winds to be expected and to currents.

For this purpose, those Ds. Lat. with respect to the Courses on the great circle's arc, as also the longitudes belonging to them and the Rhumb Course must be known.

The tracing of the two routes on the Chart would shew these comparisons most faithfully. But the Scale of Great Circles on the Blank Chart, as well as the accompanying Tables enable seamen to find with any sufficient degree of accuracy those elements without the trouble of tracing the tracks on the Chart for a preliminary investigation, supposing the qualities of the respective Rhumb line to be known.

Problem V.

To find the spherical Courses expressed in Compass Points on the great circle's arc between two given places, as well as the Lats. and Longs. in which those Courses take place.

83. After having brought the „Index-Model" into the right position upon the *Blank Chart* (Art. 33 & 37), the required Courses are on the respective curve —

those which lie on its portion between the two Index points, if the given places are on the same side of the Equator;

but, if the given places are on the opposite sides, those which lie on the two portions of that curve between each of the Index points and one of the common Intersections of the curves with the equator line; taking one of these portions between that Index point which represents the given place not transferred and that Intersection in the right position, but the other of those two portions between the Index point which represents the place transferred and that of those Intersections which is 180° distant from the first employed.

Ex. 2. Being in the entrance of the English Channel Lat. 49° 6′ N., Long. 6° 2′ W., and bound to New York, Lat. 40° 42′ N., Long. 74° 2′ W.; to find, by means of the Scale of Great Circles, the great circle Courses expressed in fourths of the Point.

The base of the Index Model is to be made equal to the D. Long. = 68° of the scale of longitudes.

After having brought the Index Model into the right position upon the Blank Chart, we find marked between the two Index points in the way from the higher latitude to the lower $6\frac{1}{2}$, $6\frac{3}{4}$, 7, &c. . . . . $7\frac{3}{4}$, 8, $7\frac{3}{4}$, &c. . . . . $5\frac{1}{2}$, $5\frac{1}{4}$, 5 Points.

The Course first denoted takes place, as an exception, at the Index point of b'', here the ship's place.

Ex. 3. To find, by means of the Scale of Great Circles, the great circle Courses expressed in fourths of the Point between Panama, Lat. 8° 57′ N., Long. 79° 31′ W., and Port Jackson, Lat. 33° 51′ S., Long. 151° 18′ E.

The Base of the Index Model is to be made equal to the supplement of the D. Long. = 50°49′. Taking the Blank Chart as the N. Hemisphere, Port Jackson is the place transferred.

After having brought the Index Model into the right position upon the Blank Chart, we find marked from the Index point of b' (Panama) to the common intersection of the curves with the equator line at the left hand (in the right position for our case) the Course $4\frac{3}{4}$ Pts.; at that Intersection the Course 53°, and from the other Intersection, that at the right hand, to the Index point of b'' (Port Jackson transferred) the Courses $4\frac{3}{4}$, 5, $5\frac{1}{4}$, &c. . . . . $7\frac{3}{4}$, 8, $7\frac{3}{4}$, &c. . . . . $6\frac{3}{4}$ and $6\frac{1}{2}$ Points. The spherical Course at Panama lies between $4\frac{1}{2}$ and $4\frac{3}{4}$ Pts., that at Port Jackson between $6\frac{1}{2}$ and $6\frac{1}{4}$ Pts. To find them exactly viz. Problem II, page 51.

Seamen who have some practice in the use of the Blank Chart may see from it exactly enough for a preliminary examination of the route on the great circle the required Lats. and Longs., referring first the longitudes of the given places which the Index lines shew on the scale of longitudes (Art. 34) to their geographical longitudes.

84. But to find exactly on the Blank Chart the Lats. and Longs. of the places at which those spherical Courses take place is less convenient than to do so *by Inspection of the accompanying Tables.*

After having found the Inclination of the great circle which passes through the two given places to the Equator and the geographical longitude of the intersection of it with the Equator (Art. 31), open the Tables at that one the No. of which is the nearest to that Inclination (if it be the intention to reduce Lats. and Longs. exactly for tracing some points on the Chart, at that Table the No. of which is equal to the quantity of whole degrees of that Inclination [page 14, (VI), &c.]). Look at this Table in the column of the latitudes for the intervals; into which the given Lats. fall. Opposite to these intervals are such in which the found Longs. from the Intersection with the Equator fall, either both into the first division (containing Longs. less than 90°), or both into the second (containing Longs. which exceed 90°), or into each division of the column of Longs. one of the two.

Then, the Courses required are to be read off as follows.

(1) If the given places are on the same side of the Equator and both the intervals are in the same division of the column of Longs.

(the track does not pass through the Vertex), take the Courses between the two intervals.

Ex. 1. To find, by inspection of the TABLES, the great circle Courses expressed in Points between St. Helena, Lat. 15° 55′ S., Long. 5° 44′ W., and C. Horn, Lat. 55° 59′ S., Long. 67° 16′ W.

The Incl. to the Equat. = 57° 18′; the Longs. fr. the Int., of St. Helena = 10° 35′ W., of C. Horn = 72° 5′ W.

Open the Table No. 57. In it we find the required Courses between the two intervals which lie between the longitudes 7° 31′ & 12° 53′ and 71° 27′ & 73° 9′, for the way from St. Helena to C. Horn, $3\frac{1}{8}$, $3\frac{1}{4}$, $3\frac{3}{8}$, &c. . . . . $6\frac{3}{8}$, $6\frac{1}{2}$ and $6\frac{5}{8}$ Points.

But if those two intervals are in this case in the different divisions of the column of longitudes (the track passes through the Vertex) take the Courses from the one of the intervals upwards to 8 Pts. and from there (8 Pts. once) downwards to the other interval.

Ex. 2. To find, by inspection of the TABLES, the great circle Courses expressed in Points between the ship's place, Lat. 49° 6′ N., Long. 6° 2′ W., and New York, Lat. 40° 42′ N., Long. 74° 2′ W.

The Incl. to the Equat. = 51° 12′; the Long. fr. the Int., of the ship's place = 68° 13′, of New York 136° 13′.

Open the Table No. 51. In it we find the required Courses from the interval which lies between the longitudes 68° 4′ & 69° 56′ to the top of the Table, $6\frac{5}{8}$, $6\frac{3}{4}$, &c. . . . . to 8 Points, and from there downwards to the interval between 135° 38 & 137° 49 longs., $7\frac{7}{8}$, $7\frac{3}{4}$, &c. . . . $5\frac{1}{4}$, $5\frac{1}{8}$ and 5 Points.

[The complete result of the solution viz. page 30, Table (1).]

(2) If the given places are on opposite sides of the Equator and both the intervals are in the same division of the column of longs. (the track does not pass through the Vertex) take the Courses from one of the intervals to the bottom of the Table, the Course at the point of intersection of the great circle with the Equator and the Courses from there to the second interval.

But if the two intervals are in the different divisions of the column of longitudes (the track passes through the Vertex) take the Courses from the interval in the first division downwards to the bottom of the Table, the Course at the point of intersection of the great circle with the Equator, the Courses from there to the top of the Table, and then downwards to the other interval.

Ex. 3. To find, by inspection of the TABLES the spherical Courses expressed in Points between Panama, Lat. 8° 57′ N., Long. 79° 31′ W., and Port Jackson, Lat. 33° 51′ S., Long. 151° 18′ E.

The Incl. to the Equat. = 37°; the Long. fr. the Int., of Panama = 12° 4′ E., of Port Jackson = 117° 7′ W.

Open the Table No. 37. In it we find the Courses required from the interval between the longitudes 16° 54′ & 8° 11′ to the bottom $4\frac{3}{4}$ Pts., at the point of Inters. of the great circle with the Equator = 53°, from the bottom to the top $4\frac{3}{4}$, $4\frac{7}{8}$, 5, &c. . . . . $7\frac{3}{4}$, $7\frac{7}{8}$ and 8 Points, and from there to the other interval, that between the longitudes 116° 8′ & 118° 15′, $7\frac{7}{8}$, $7\frac{3}{4}$, &c. . . . . $6\frac{7}{8}$, $6\frac{3}{4}$ and $6\frac{5}{8}$ Points.

The Equatorial Tracks, the Polar Tracks and their Limits.

85. Besides the circumstance that the whole great circle track between two given places or parts of it are not navigable, prevailing winds or currents may be the cause for chosing a track different from that on the great circle. The Rhumb line as the generally known track, as the track most convenient on account of its determination, forms the base for such circuitous tracks with respect to the Pattern track, the great circle's arc between the two given places. Supposing both these tracks to be known (traced or imagined on the Chart), and their Ds. Lat., each in the same Long. (on the same meridian), to be divided into any one and the same quantity of equal parts, the tracks passing through the homologous points (in alike positions) save the more distance the nearer they are to the great circle's track.

Because the Rhumb line between two given places lies, at the common beginning of the tracks at each of the given places, always towards the Equator with respect to the respective great circle's arc,

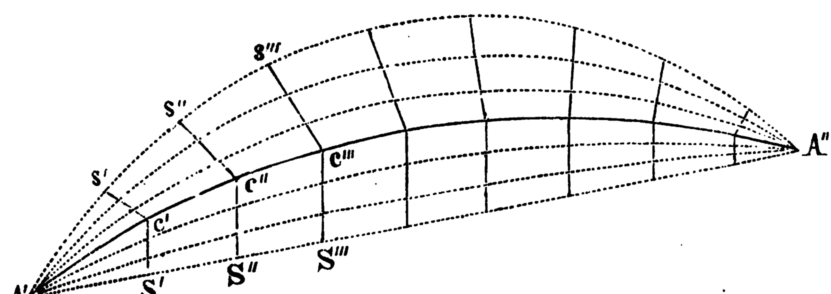
we shall call those intermediate tracks ***Equatorial Tracks***, and the Rhumb line, the ***Limit of the Equatorial Tracks.***

Supposing the prevailing wind to be a foul wind with respect to the route on the great circle's arc and to be contrary to the direction of the Rhumb line, the ship may sail free shaping the course either from the Rhumb line towards the Equator or on the polar side of the great circle. She Seaman who in this case, would shape the course on the side of the Rhumb line towards the Equator may conduct his ship away from her place of destination instead of nearing it. This causes a determination of a series of tracks on the polar side of the great circle's arc like the Equatorial tracks on its side towards the Equator.

We shall call such tracks „***Polar Tracks***“, and that of them which is equal in length to the Rhumb line „***the Limit of the Polar Tracks***“. Its determination is completely explained Art. 65 -- 74. Such limits of polar tracks are drawn on the ***Title Charts*** with curves dotted in the same manner as the respective Rhumb lines and one of them, that between a place in the entrance of the English Channel and New York is exactly illustrated by the table No, 5, page 62. The employing of a portion of this Limit which is still less navigable in its whole extent, than the great circle's arc between the two places adopted as given Ex. 2., is made Art. 92, page 77.

The polar tracks between their Limit and the great circle's arc are determined by the homologous points on the distances between the points on that limit and the respective great circle's points (Art. 67) dividing them into a quantity of equal parts, just as in determining points of the equatorial tracks dividing the respective Ds. Lat. Each of these intermediate polar tracks is the shorter, the nearer it is to the great circle's arc.

Let A' & A'' be two given places. $A'S'S'' \ldots A''$ the Rhumb line, $A'c'c'' \ldots A''$ the great circle's arc, and $A's's'' \ldots A''$ the limit of the polar tracks between them,



then the two curves drawn between the great circle's arc and the Rhumb line shew equatorial tracks and the curves between that circle's arc and the limit of the polar tracks shew polar tracks between the two given places.

II.

THE PRACTICAL STAGES FOR SHAPING THE COURSE ON A GREAT CIRCLE.

86. In case the route of a ship is, as an exception, pointed out by remarkable fixed objects, the seaman takes care to conduct the ship in the vertical plane designed by those signals, and thus he takes truly care to steer exactly on an arc of a great circle (Art. 17), to steer directly the shortest distance (Art. 19). During such a steering, to watch the necessary Changing of the Courses by the coinciding of the points of the compass-card with the Lubber-point is not to be expected, although to observe the differences of the terrestrial Azimuths at different points on the route measured with an Azimuth Compass may be possible, if the portion of the great circle along which the ship is steering differs visibly from the respective Rhumb line.

To steer in the open Sea along a great circle's arc is only to be hoped for, if also there the route is marked, which can never be done except by representing the respective great circle's track on the Chart. But, because in practice it is necessary to have a definite direction for a distance in which the portion of the great circle's arc does not differ in length from the respective Rhumb line, the route on the great circle is to be broken up into stages *) which are to be taken as long as possible for corresponding to that condition.

Mercator's projection represents the Azimuth of a terrestrial line at each point of it in its true magnitude, that is, the spherical Course at a place on a great circle is represented on the Chart by the straight-lined angle made by the meridian of that place and the tangent drawn through the place to the curve which represents that circle.

To investigate the degree of the difference in length between the curve which represents a great circle on the Chart and the Rhumb line which is a tangent to that curve reckoning their length from the point of contact, we can compare the respective portion of the curve with an arc of a circle described on the plane of the Chart, and the Rhumb line with the side of a polygon described about that circle.

87. In any rectilineal figure the sum of the exterior angles is equal to four right angles and in a regular polygon the exterior angles are equal to one another, wherefore in each of such polygons the sum of its sides is equal to the quotient dividing four right angles (32 Points) by the measure of its exterior angle.

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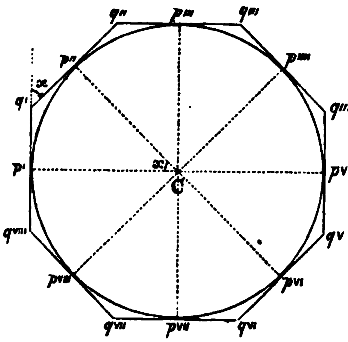
*) The usual instruction to change the Course from time to time or at short distances is not sufficient by the experience of practical seamen. It leads to an error, if the Navigator takes at the end of those stages the spherical Course corresponding to those distances or to the respective Longs.. The ship is in those cases conducted first on a tangent of the projection of the great circle on the Chart, but after that, in a broken line which lies the more removed from the great circle's track the farther she goes in this manner. To trace on the Chart only the point of *Maximum Separation* and the Vertex of the circle, when there is one, gives as little the surety for shaping the course on the great circle.

The problem „to find the point of *Maximum Separation in Latitude*“ (of the great circle's arc and the respective Rhumb line) is solved by employing Art. 51, substituting the Rhumb Course as the Course adopted at a point of the great circle.

Fig. 29.



In any regular polygon described about a circle, the exterior angle is equal to the angle made by any two radii which join the centre with two successive points of contact. In Fig. 29 the exterior angle x is equal to the angle x at the centre.

The number of those angles at the centre is equal to the number of the sides of the polygon and their sum is always equal to four right angles. Dividing four right angles by the number of the sides gives therefore the measure of the angle x at the centre as also of the exterior angle x; and *vice versa*, dividing 32 Points by the exterior angle expressed in Points gives the number of the sides of the polygon described about the circle.

Adopting the diameter of the circle as unit, the elementary calculation of the circumferences of regular polygons described about the circle gives the following results.

If the number of the sides of the polygon is equal to	the exterior angle of the polyg. is equal to	the circumference of the polyg. is equal to
4	8 Points	4·0000
8	4 „	3·3137
16	2 „	3·1826
32	1 „	3·1517
64	$\frac{1}{2}$ „	3·1441
128	$\frac{1}{4}$ „	3·1422
256	$\frac{1}{8}$ „	3·1418

The circumference of the circle, adopting its diameter also as unit of the measure, is equal to . . . . 3·1416

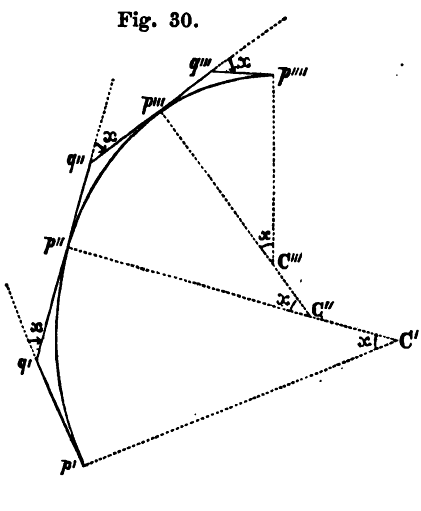
Thus, the ratio between the circumference of a regular 128 sided polygon, described about a circle, and the circumference of the circle is equal to 3·1422 to 3·1416, equal to $\frac{31422}{31416} = 1{\cdot}0002$. The same ratio will take place between the sum of two half sides of such a polygon, meeting each other at an angle of which the supplement is equal to $\frac{1}{4}$ Point, and the circle's arc between the points of contact of those sides.

If the angle x (Fig. 29) is equal to $\frac{1}{4}$ Point, then $(p'q' + q'p'')$ is to the arc $p'p''$ as 3·1422 is to 3·1416.

The difference in length between the sum of such two half sides of the polygon described about the circle and the respective arc of the circle does therefore not require to be considered in practice.

88. If the points p', p'', p''', &c. (Fig. 30) are adopted on any curve in a manner that tangents drawn through them to the curve meet each other at angles of which the supplements are equal to the same angle x adopted very small, each of the sums of the adjacent parts of those tangents from their point of inters. to their points of contact $(p'q'+q'p'')$, $(p''q''+q''p''')$, &c., is to be considered as the sum of two adjacent half sides of a regular polygon the exterior angle of which is equal to x and which is described about a circle of which the portion of the curve between the respective two points of contact is an arc, $p'p''$, $p''p'''$, &c., and of which the centre is, respectively C', C'', &c.

Fig. 30.



Adopting the angle $x = \frac{1}{4}$ Point, the sums of the adjacent parts of the tangents, $(p'q' + q'p'')$, $(p''q'' + q''p''')$, &c., are to be compared with the respective parts of the sides of such 128 sided polygons, and therefore, each of those sums does in practice not differ in length from the length of the portion of the curve between the two respective points of contact.

Applying this to the determination of practical stages for shaping the Course on a great circle we have,

„places on the great circle between which, the Changing of the „Courses amounts to $\frac{1}{4}$ Point are extremities of portions of the „great circle which in practice does not differ in length from the „broken Rhumb track between those two places, compounded „of the two Rhumb lines each of which touches the projection „of the great circle on the Chart in one of those places.“

89. The accompanying Tables to facilitate the practice of Great Circle Sailing (Descr. pag. 9, Applic. Art. 84) as well as the Scale of Great Circles on the Blank Chart (Descr. page 41, Applic. Art. 83) are founded upon this principle, shewing the Latitudes and Longitudes from the Intersection with the Equator on the great circle of the globe corresponding to each $\frac{1}{4}$ Point of Course.

But, although the whole distance on the broken Rhumb track would be determined to a sufficient degree of accuracy by taking for it the respective spherical distances, the two Rhumb lines, $p'q'$ & $q'p''$, which form that track are not so exactly equal to one another as on a regular polygon, because they cannot be compared with tangents to the same true circle described on the Chart.

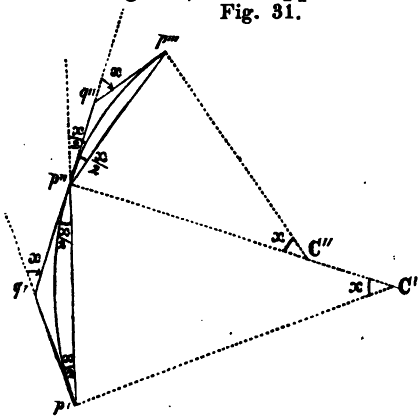
Joining $q'C'$ in Fig. 30, this straight line would not exactly bisect the portion $p'p''$ of the curve, and the parts cut off by it would have different centres belonging to arcs of circles described on the plane of the Chart.

To determine the distances on the respective Rhumb lines in which the Course is to be changed by $\frac{1}{4}$ Point, the places on the great circle at which the Course corresponds to each $\frac{1}{8}$ Point are intermediate in the Tables. Every one of the spherical distances between each of the two points of contact and that intermediate place is equal to the required distance on the respective touching Rhumb line.

Viz. page 30, (2), Extract of the Stages which are tangents on the Chart.

90. Let p' & p'' or p'' & p''' (Fig. 31) be two points on any curve and the angle x, the supplement of the angle made by tangents to the curve drawn through those points, be small enough to compare the portion of the curve between the two points with an arc of a circle described upon the plane of the curve, then drawing the chords $p'p''$, $p''p'''$, such a chord makes, with the respective two tangents, angles equal to $\frac{x}{2}$, because the triangles $p'q'p''$, $p''q''p'''$, are isosceles triangles; and then, the supplement of the angle made by two adjacent chords, $p'p''$ and $p''p'''$, is equal to $\frac{x}{2} + \frac{x}{2} = x$.

Fig. 31.



Adopting the angle $x = \frac{1}{4}$ Point, each of the chords $p'p''$, $p''p'''$, is to be compared with the side of a 128 sided regular polygon inscribed in a circle the centre of which is as before, respectively, C', C''; wherefore each of those chords in practice does not differ in length from the portion of the curve between the respective two points, and the supplement of the angle made by two adjacent chords is very nearly equal to $\frac{1}{4}$ Point.

For the application of the accompanying Tables to facilitate the practice of great circle sailing, it follows from the above.

(1) When the ship's place is any one of the places on a great circle determined by the Tables, steering along a single Rhumb line from that place to either of the second next determined in the respective table (to one of the two places at which the changing of the Course on the great circle amounts to $\frac{1}{4}$ Pt.),

the length of that Rhumb line does not differ in practice from the length of the great circle's arc between those two places; and

that Rhumb Course is very nearly $\frac{1}{8}$ Pt. greater than the spherical Course indicated in the Table, if steering towards a place in higher Lat.; it is very nearly $\frac{1}{8}$ Pt. smaller than that spherical Course, if steering towards a place in lower Lat.

(2) When steering along several such single Rhumb lines in succession, the Course changes very nearly by $\frac{1}{4}$ Pt. at the ends of each of such Rhumb tracks, places on the great circle.

Viz. page 30, (3), Extract of the Stages which are Chords on the Chart.

III.

COMPOUND SHIP ROUTES.

91. The route on a great circle's arc as well as the route on an equatorial track or on a polar track between two given places reduced to a broken line of Rhumb tracks determined by applying the accompanying TABLES will not be considered by us as a compound route.

In investigating great circle sailing, it is the custom to take the position of the port of departure and that of the port of destination from the *Maritime Positions* as given places between which different ship tracks are to be compared, even if those tracks are impracticable at short distances from the respective port, supposing that in practice, when the ship after her departure has got into the open sea, and circumstances permit, the Course is to be shaped with respect to a great circle which passes through that ship's place and any accessible place as near as possible to the other port. The difference between the comparison of the different tracks belonging to these places free of the land with those belonging to the respective ports themselves will be insignificant.

For instance. Ex. 2 is litterally borrowed from *The Great Circle Tables for the North Atlantic (page* 18; *Ex.* 1*) by James Masters Share* *) to facilitate the comparison between the result by applying those tables to determine the „*Polar Course*“ and our determination of the „*Limit of the Polar Tracks*“. The circumstance that the routes constructed in this example are not practicable for a short distance from New York on account of Long Island is not considered in what follows.

Ex. 3 is borrowed from *Mr. Towson's generally known Tables for Great Circle Sailing* **) *(Page* 41, *Ex. IV).* It is not considered in the following that, in practice, it is necessary to determine the tracks, instead of from Panama, from a place near Panama at the entrance of Panama Bay for passing freely Mala Point.

92. If for a long voyage from a place on one of the coasts of the North Atlantic, the great circle passing through that place and the port of destination passes through the continent of Africa or through that of America, it is evident that the Course is to be shaped with respect to at least two adjacent great circle's arcs for which the intermediate place lies, respectively, near the Cape of Good Hope or near Cape Horn, or with respect to great circle's arcs joined by a track passing that Cape. The route of a ship at a place near Cape Breten bound to Port Adelaide may be projected with respect to a single great circle's arc which passes the Cape of Good Hope. But, besides that this arc conducts the ship in the Indian Ocean in latitudes which are not practicable in all seasons (Art. 93), it may happen that cutting the Equator and having passed the Cape of Good Hope, the Course is to

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*) London. Charles Wilson, Late J. W. Norie and Wilson, 157, Leadenhall Street 1852.

**) Fourth Edition. London, Printed for the Hydrographic Office, Admiralty; and sold by J. D. Potter, Agent for the Admiralty Charts, 31, Poultry 1852.

be shaped anew with respect to arcs of great circles which differ more or less from that which passes through the two given places.

Such cases as well as such in which the route between the two given places is composite of a curve referred to a great circle's arc, and of a proper Rhumb line, are to be solved completely by applying rules given in the preceding, if the application of Art. 93 would not be requisite.

But, if neither the route on the great circle's arc between two given places nor the polar tracks between them are practicable in their whole extentions, and an intermediate place determines the two tracks which give the shortest practicable route between the two given places as a compound route, portions of those single tracks between the two given places are not to be excluded — in the choice of the routes with respect to prevailing winds — forming the compound tracks.

For instance.

Ex. 2 (Being in the entrance of the English Channel, Lat. 49° 6′ N., Long. 6° 2′ W., and bound to New York, Lat. 40° 42′ N., Long. 74° 2′ W.) is theoretically solved to illustrate the different investigations and rules given in the preceding. The most simple treatment of this example (a comparison of the curve of the Scale of Great Circles belonging to the Lat. of Vertex of 51° 12′ with the Chart; or, opening the Table No. 51, the Lats. there found for the route with respect to their geographical Longs. united with an inspectional view of the Chart; or, an application of Mr. R. Russel's Diagram *); or, turning a Terrestrial Globe in a manner that the upper edge of its broad paper circle passes through the two given places; or, stretching a thread between the two given places upon the surface of such a globe) would have shewn that the great circle's arc as well as the Limit of the polar tracks between the two given places does not give a route practicable in its whole extension, and that Cape Race is the intermediate place for determining the compound tracks between the two given places.

The practical seaman will find out between the *five routes* illustrated on the right hand page and there arranged with respect to their Courses, that route on which he may conduct his ship with respect to prevailing winds in the shortest time to the port of destination.

Remarks. 1. The great circle passing through the ship's place and C. Race Lat. 46°40′, Long. 53°7′, has an inclination to the Equator = 50°30′. The Long. of its Intersection with the Equator = 66°1′ E. by which the TABLE's Long. and the geographical Long. of any place on the great circle differ from each other.

The illustration of its arc between those two places which is here given is made by inspection of TABLE No. 50, as the table (3), page 30, is made by inspection of TABLE No. 51. (Use of the TABLES, (V), page 13; & Art. 90.)

Neither the Rhumb line nor the great circle's arc between the ship's place and C. Race are drawn on the Title Chart No. 1 not to crowd that Chart with a multiplicity of lines.

2. The Rhumb line between C. Race and New York produced beyond C. Race (viz. Title Chart No. 1) terminates, meeting the great circle's arc between the ship's place and New York, and the limit of the polar tracks between these two places, the greatest portions of each of these curves which may be taken as parts of the respective compound routes. The parts of those curves employed in the illustrations here given are extracted, respectively, from the table (3), page 30, and the table No. 5, page 62. On the portion of that great circle, a point determined by the table lies convenient, but on the track on the limit of the polar tracks, the distance on the last stage is made 100 miles, because that last given (table No. 5, page 62), in length 148 miles, would lead the ship beyond the Rhumb line between New York and C. Race produced. — The Lat. and Long. of the end of that stage of 100 miles are found by Mercator's Rule.

3. The compound track composite of the limit of the polar tracks between the ship's place and C. Race and the Rhumb line between C. Race and N. York is not received in the Illustration here given.

The distance on this route would be like that on the two respective Rhumb lines = 2878, but the Course of its first portion would not differ so such from that on the great circle's arc between the ship's place and C. Race as the Course on the great circle's arc between the ship's place and New York differs from that great circle's arc.

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*) Viz. Note **) page 53.

Course	Lat. °	′	Long. fr. Greenw. °	′	Dist. Miles
The Rhumb line from the ship's place to N. York					
Course	49	6	6	2	
S. 80° 5′ W.					2927
	40	42	74	2	
Distance on the whole route				Sum	2927

Course	Lat. °	′	Long. fr. Greenw. °	′	Dist. Miles
The Rhumb line from the ship's place to Cape Race					
N 85° 36′ W	49	6	6	2	
					1903
	46	40	53	7	
The Rhumb line fr. C. Race to N. York					
S 68° 27′ W	46	40	53	7	
					975
	40	42	74	2	
Distance on the whole route (Sum)					2878

Course	Lat. °	′	Long. fr. Greenw °	′	Dist. Miles
The great circle's arc between the ship's place and C. Race					
N 77° 7′ W	49	6	6	2	
					58
W b N (N 78° 37′ W)	49	19	7	29	
					147
W $\frac{3}{4}$ N (N 81° 57′ W)	49	48	11	11	
					143
W $\frac{1}{2}$ N (N 84° 19′ W)	50	8	14	51	
					141
W $\frac{1}{4}$ N (N 87° 7′ W)	50	22	18	31	
					140
West (90°)	50	29	22	10	
					139
W $\frac{1}{4}$ S (S 87° 7′ W)	50	29	25	48	
					140
W $\frac{1}{2}$ S (S 84° 19′ W)	50	22	29	27	
					141
W $\frac{3}{4}$ S (S 81° 57′ W)	50	8	33	7	
					143
W b S (S 78° 37′ W)	49	48	36	47	
					147
W b S $\frac{1}{4}$ S (S 75° 48′ W)	49	19	40	29	
					151
W b S $\frac{1}{2}$ S (S 73° 21′ W)	48	42	44	12	
					157
W b S $\frac{3}{4}$ S (S 70° 6′ W)	47	57	47	58	
					164
S 69° 0′ W	47	1	51	47	
					59
	46	40	53	7	
The Rhumb line fr C. Race to N. York					
S 68° 27′ W	46	40	53	7	
					975
	40	42	74	2	
Distance on the whole route (Sum)					2845

Course	Lat. °	′	Long. fr. Greenw. °	′	Dist. Miles
Route on the great circle between the ship's place and New York					
N 73° 12′ W	49	6	6	2	
					73
W b N $\frac{1}{4}$ N (N 75° 51′ W)	49	27	7	48	
					147
W b N (N 78° 43′ W)	50	3	11	29	
					143
W $\frac{3}{4}$ N (N 81° 47′ W)	50	31	15	9	
					139
W $\frac{1}{2}$ N (N 84° 33′ W)	50	51	18	47	
					138
W $\frac{1}{4}$ N (N 87° 4′ W)	51	4	22	24	
					136
West (90°)	51	11	26	1	
					136
	51	11	29	37	
The Rhumb line to Cape Race					
S 73° 41′ W	5[illegible]	11	29	37	
					965
	46	40	53	47	
The Rhumb line fr. C. Race to N. York					
S 68° 27′ W	46	40	53	47	
					975
	40	42	74	2	
Distance on the whole route (Sum)					2852

Course	Lat. °	′	Long. fr. Greenw. °	′	Dist. Miles
Route on the Limit of the Polar Tracks between the ship's place and N. York					
N 46° 39′ W	49	6	6	2	
					71
N 50° 58′ W	49	55	7	22	
					149
N 57° 4′ W	51	29	10	25	
					149
N 62° 22′ W	52	50	13	49	
					149
N 68° 40′ W	53	59	17	30	
					100
	54	36	20	9	
The Rhumb line to Cape Race					
S 69° 10′ W	54	36	20	9	
					1338
	46	40	53	47	
The Rhumb line fr. C. Race to N. York					
S 68° 27′ W	46	40	53	47	
					975
	40	42	74	2	
Distance on the whole route (Sum)					2931

93. With respect to the formation of all kinds of Compound Ship's Routes we have here only to treat anew that which is usually known under the name of

Composite Great Circle Sailing.

It includes the cases in which the route on the great circle's arc between the two given places is not practicable in the highest latitudes through which it passes, but the route on a certain Parallel which divides the great circle's arc into its practicable and impracticable parts is practicable. The latitude of this Parallel in usually called the „*Maximum Latitude*".

The employing of a polar track between the two given places will be almost generally excluded *). Not so the equatorial track (Art. 85) which touches the parallel of the Maximum Lat., but the following enables seamen to construct a route which is shorter than that equatorial track and which is in our cases the shortest possible route between the two given places.

This track to be formed instead of the great circle's arc between the two given places consists, if neither of the places lie in the maximum Lat., of *three adjacent arcs* of which the two exterior belong to great circles of which the Lat. of Vertex (Inclination to the Equator) is equal to the adopted Maximum Lat., and of which the middle arc belongs to the parallel circle of the Maximum Lat. This circle touches each of the two great circles externally on the surface of the globe. If the Lat. of one of the given places determines the Maximum Lat., the track to be formed consists of *two adjacent arcs* of circles which touch each other also externally on the surface of the globe, and of which the one is the great circle the Lat. of Vertex of which is equal to the Maximum Lat., the other is the parallel circle of that place which determines the Maximum Latitude.

If a great circle and a parallel circle touch each other on the surface of the globe, the point of contact is always the Vertex of the great circle, wherefore tracing the route on the Chart, if neither of the given places lies in the Maximum Lat., we have twice to draw the curve which represents the great circle the Lat. of Vertex of which is equal to the Maximum Lat., passing separately through the given places, from each of the places to the Vertex of the curve, and to connect the two Vertexes by a straight line coinciding with the Parallel of the Maximum Latitude.

The Title Chart No. 2 shews besides the Rhumb line and the great circle's arc between a place near the Cape of Good Hope and a place near Port Adelaide, two such compound routes, for one of which 50°, for the other 46° are adopted as Maximum Latitude.

To trace this kind of compound routes on the Chart, if the Lat. of one of the two given places determines the Maximum Lat., draw the part of the curve which represents the great circle of which the Lat. of Vertex is equal to that Maximum Lat. passing through the other given place from that place to the Vertex of the curve, and then draw a straight line from that Vertex to the place by which the Maximum Latitude is determined.

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*) Always if the Parallel of the Maximum Latitude is the Limit of the region not free of ice &c.; even it may be so, if an island makes the great circle's track impracticable, viz. Ex. 3 the great circle's arc between Panama and Port Jackson belonging to the island of New Zealand, Fig. pag. 80.

The Title Chart No. 1 shews besides the Rhumb line and the great circle's arc between a place near the Cape of Good Hope and a place near Cape Horn such a compound route adopting the Lat. of that place near Cape Horn as Maximum Latitude.

94. The application of the Scale of Great Circles as well as of the accompanying Tables to the different cases of Composite Great Circle Sailing is after what has been said on this sailing and in general on the use of that Scale and of those Tables easy to be made, having solved the following problem with respect to each of the given places, or to only one of them, according as a great circle passes through both or only through one of these places.

Problem VI.

Having given the Inclination of a great circle to the Equator (Lat. of Vertex) as the Maximum Lat., and the Lat., b, as well as the geographical Long. of a place on that great circle, to find the place's Long. from the Intersection of the circle with the Equator (Table Long.) and the geographical Long. of that Intersection.

By Inspection

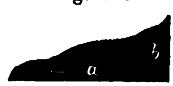
by means of the Scale of Great Circles on the accompanying Blank Chart.

To make the „Index Model".

Cut off two of the sides of a piece of thick paper to straight lined edges right angled to each other. Place one of these edges along the equator line, the other along the meridian of one of the common intersections of the curves, and mark on this edge the given latitude, *b*, from the adjacent scale of latitudes.

Cut off the paper in such a manner as Fig. 32 shews, so that the extremity of *b*, the only Index point, be represented as point of an acute angled corner and the other straight lined edge, the base *a*, remains long enough to judge of its exact coinciding with the equator line in the following operation.

Fig. 32.



To dispose the Index Model's edge *b* as Index upon the Blank Chart.

Place the Index Model upon the Blank Chart so that its base, *a*, coincides with the equator line and the Index point (the extremity of *b*) hits the curve the No. of which is equal to the given Maximum Lat., the Inclination of the great circle to the Equator (Lat. of Vertex).

(In most cases, the Maximum Lat. will be adopted in whole degrees and then that curve is printed on the Blank Chart. If the Maximum Lat. be determined by the Lat. of a given place, that curve might be one of the not printed and is to be imagined between the two of which the one is numbered like the quantity of whole degrees of that Lat., the other numbered by one higher.)

In this position of the Model the Index line shews directly on the scale of longitudes the required *Long. from the intersection of the great circle with the Equator (Tables Long.)* of the given place.

Subtracting algebraically this Long. from the given Long. of the place gives the required geographical Long. of the Intersection of the great circle with the Equator.

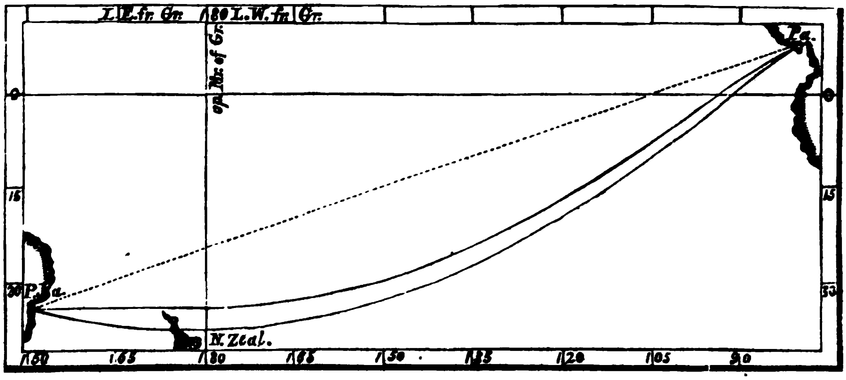
By Calculation.

(Solution of the right-angled spherical triangle; Appendix II, formula 4.)

To the log. cot. of the Max. Lat. add the log. tan. of the Lat. of the given place; the sum (rejecting ten) is the log. sine of the required long. from the Inters. of the great circle with the Equator (Tables Long.).

Subtract algebraically this Long. from the given Long. of the same place, the difference is the required geographical Long. of the Inters. of the great circle with the Equator.

Ex. 3. To determine by calculation the shortest practicable ship's route between Panama, Lat. 8°57′ N., Long. 79°31′ W. and Port Jackson, Lat. 33°51′ S., Long. 151°18′ E.



The shortest line between the two given places, the great circle's arc between them, is completely determined in the preceding. The distance on it is found equal to 7633 miles.

But a less exact treatment of the example would already have shewn that this great circle's arc passes through the island of New Zealand and that the Parallel of the Lat. of P. Jackson passes the Three Kings, Lat. 34°13′ S., Long. 172°10′ W., northwards, wherefore the shortest practicable route is a compound track composite of an arc of the great circle, passing through Panama, the inclination of which to the Equator is determined by the Lat. of P. Jackson as Maximum Latitude and of the portion of that Parallel between the Vertex of that great circle and Port Jackson.

Adopting the Maximum Lat. = 33°51′,

to find Panama's Long. from the Intersection of the great circle with the Equator.

Maximum Lat.	33°51′	log. cot.	10·1735
Lat. of Panama	8°57′	log. tan.	9·1973
Lat. req. of Pa. 13°25′		log. sin. (sum rej. 10)	9·3708

to find the geographical Long. of the Intersection of the great circle with the Equator.

Panama's geographical Long.	79°31′ W.
its Long. fr. the Inters.	13°25′ E.
geogr. Long. required (algebraical diff.)	92°56′

To find the Dist. on the whole route.

	Diff. Long. of each part.		Dist. on each part.
1. Between Panama and the Intersection	= 13°35′	— (Rule, Art. 62)	973 miles
2. between the Inters. and the Vertex	= 90 0	—	5400 „
between Panama and the Vertex (sum)	103°35′		
D.Long. betw. Panama and P.Jackson	= 129°11′		
between Panama and the Vertex	= 103 35		
3. between the Vertex and P. Jacksen (diff.)	25°36′	— (log.D.Lg.+log.cos.Max.Lat.=log.Dist.)	1274 „
		Dist. on the whole route (sum)	7647 miles

Remark. The Dist. on the great circle passing through Panama and P. Jackson from Panama to the Parallel of P. Jackson and farther on this Parallel to P. Jackson is equal to 7665 miles.

IV.

REMARK ON WINDWARD GREAT CIRCLE SAILING.

95. The route on a great circle's arc, as well as the route on an equatorial track, the route on a polar track and the compound route is reduced by the preceding to a broken Rhumb track of which the single Rhumb lines are determined in every respect, and of which the Dist. on the shortest which may be possible in practice (near the Vertex in high latitudes) is 130 miles; wherefore Great Circle Sailing is really reduced to Rhumb Sailing by the principle adopted and advocated by us.

When on account of adverse winds windward sailing is requisite for a time, the rule for it is no other, than that generally known with respect to the Rhumb line; in our case any stage on the great circle's route.

Only we take here the opportunity to notice that, if the stages of the broken Rhumb line are chords on the Chart, one of them might be produced to the next tangent produced backwards (Fig. 31, $p'p''$ to $p'''q''$ produced), or, if those stages are tangents on the Chart (Fig. 30), one of them might be produced to the next chord produced backwards (Fig. 31, $p'q'$ to $p'''p''$ produced) without increasing the distance considerably, taking the course of the new kind of stages, when meeting it produced. As well as, that circumstances may make it profitable, to miss one or two stages which are chords on the Chart, taking the Rhumb Course to the end of the second or third following stage and going from there again along the great circle's arc between the two given places.

APPENDIX.

NOTE

ON THE DETERMINATION OF AZIMUTHS, WITHOUT CALCULATION.

Mr. J. T. Towson gives, besides the Explanation of his generally known Linear Index and Great Circle Tables to facilitate the practice of great circle sailing, an Instruction on their Application to find without calculation the Azimuth of a Heavenly Body.

Our New Great Circle Tables are only constructed to facilitate the practice of great circle sailing, to make this sailing itself most convenient reducing it to a broken track of which each stage is a Rhumb as long as possible without making a circuitous route, as well as to make also most convenient the comparison of the theory of the route on a great circle's arc with any other kind of ship-routes.

For this purpose the spherical Courses (terrestrial Azimuths) in those Tables as well as on the accompanying Scale of Great Circles are adopted as quantities to be expressed by the Compass Point and fractions of it. These Tables as well as that Scale are therefore not qualified to determine an azimuth of a heavenly body by them.

But Art. 38 — 42 and IV. of the Appendix, together with Art. 54 — 56, contain the solution of the

PROBLEM

to find by construction the Azimuth of a Heavenly Body, having given the Lat. of the ship's place, as well as the declination and the hour-angle of the heavenly body;

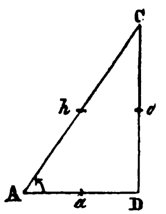
substituting, Fig. 20 and 25, for a, the hour-angle of the heavenly body;
for b', the Lat. of the ship's place
or, the declination of the heavenly body;
and for b'', respectively, the declination of the heavenly body
or, the Lat. of the ship's place;
Fig. 25, for the angle A', instead of the Course, the Azimuth (its supplement) of the heavenly body, if b' is the Lat. of the ship's place;
but for the angle A'', instead of the Course, the Azimuth (its supplement) of the heavenly body, if b'' is the Lat. of the ship's place.

I.

SOME PROPOSITIONS OF PLANE TRIGONOMETRY.

SOLUTION OF THE RIGHT-ANGLED PLANE TRIANGLE.

1. In measuring the sides of any right-angled plane triangle with any one but the same unit of length, the numbers which represent the sides have the *same denomination,* that of the unit employed, wherefore the ratios between any two of them may be expressed by an abstract fraction.



Let the number which represents the hypotenuse *AC* of the right-angled triangle *ADC* be equal to *h*, that which represents the leg adjacent to the angle *A*, *AD*, equal to *a*, and that which represents the leg opposite to the angle *A*, *CD*, equal to *o*, all three lines being determined by the same measure:

then *supposing the Trigonometrical Ratios to be known* we have,

$$\frac{o}{h} = \sin A, \ \therefore o = h \times \sin A \qquad \frac{h}{o} = \operatorname{cosec} A, \ \therefore h = o \times \operatorname{cosec} A$$

$$\frac{o}{a} = \tan A, \ \therefore o = a \times \tan A \qquad \frac{a}{o} = \cot A, \ \therefore a = o \times \cot A$$

$$\frac{h}{a} = \sec A, \ \therefore h = a \times \sec A \qquad \frac{a}{h} = \cos A, \ \therefore a = h \times \cos A$$

multiplying both the sides of each of the first equations by the divisor on its left hand side.

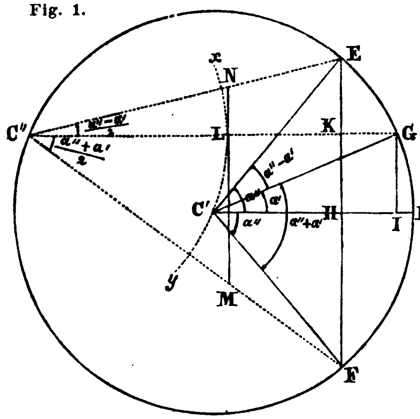
ELEMENTARY DEMONSTRATION OF THE TWO FORMULÆ.

$$(1)\ (\sin a'' + \sin a') : (\sin a'' - \sin a') :: \tan\frac{a''+a'}{2} : \tan\frac{a''-a'}{2} \text{ or, } \frac{\sin a'' + \sin a'}{\sin a'' - \sin a'} = \frac{\tan\frac{a''+a'}{2}}{\tan\frac{a''-a'}{2}}$$

$$(2)\ (\tan b'' + \tan b') : (\tan b'' - \tan b') :: \sin(b''+b') : \sin(b''-b') \text{ or, } \frac{\tan b'' + \tan b'}{\tan b'' - \tan b'} = \frac{\sin(b''+b')}{\sin(b''-b')}.$$

2. Let the angles *DC'E* and *DC'F* (Fig. 1) be equal to each other and denoted by a'', and the angle *DC'G* be denoted by a', then the angle *FC'G* represents the sum of a'' and a', $(a'' + a')$, and the angle *EC'G* the difference between a'' and a', $(a'' - a')$.

Fig. 1.



Describe from *C'* as centre with the radius *C'D* the circle *DGEC''FD*; draw the chord *EF* which crosses *C'D* in *H* and is bisected by *H*; draw from *G* the line *GI* perpendicular to *C'D*, and through

G the line GC'' parallel to $C'D$ which crosses EH in K and cuts off $KH = GI$. Then the following sines are represented with respect to the radius of the circle described: —

$$\begin{aligned} \sin a'' &= FH \\ \sin a' &= GI = KH \\ \therefore \sin a'' + \sin a' &= FH + KH = FK \end{aligned}$$

$$\begin{aligned} \sin a'' &= EH \\ \sin a' &= GI = KH \\ \therefore \sin a'' - \sin a' &= EH - KH = EK \end{aligned}$$

Join $C''E$ and $C'F$, then the angle $FC''G$ is equal to half the angle $FC'G$, as also the angle $EC''G$ is equal to half the angle $EC'G$ (because the angle at the centre = twice the angle at the circumference, both upon the same arc), wherefore

$$FC''G = \frac{a'' + a'}{2},$$

$$\text{and } EC''G = \frac{a'' - a'}{2}.$$

Describe from C'' as centre with a radius equal to $C'D$, the radius of the circle first described, the arc xy which crosses $C''G$ in L, and draw through L the line NM parallel to EF, then it is perpendicular to $C''L$, and the following tangents are represented with respect to the radius $C''L = C'D$,

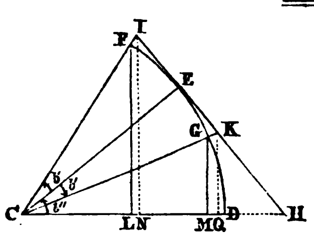
$$\tan\frac{a'' + a'}{2} = LM$$

$$\tan\frac{a'' - a'}{2} = LN.$$

But, because NM is parallel to EF, the triangle $C''KF$ is similar to $C''LM$, and $C''KE$ similar to $C''LN$, and we have

$$\begin{array}{rcccccc} FK & : & LM & :: & C''K & : & C''L \\ \text{and } EK & : & LN & :: & C''K & : & C''L \\ \therefore FK & : & LM & :: & EK & : & LN \\ \text{or } FK & : & EK & :: & LM & : & LN \end{array}$$

$$\left.\begin{array}{l} \text{that is } (\sin a'' + \sin a') : (\sin a'' - \sin a') :: \tan\dfrac{a'' + a'}{2} : \tan\dfrac{a'' - a'}{2} \\[2ex] \text{or } \dfrac{\sin a'' + \sin a'}{\sin a'' - \sin a'} = \dfrac{\tan\dfrac{a'' + a'}{2}}{\tan\dfrac{a'' - a'}{2}}. \end{array}\right\} \quad \ldots\ldots (1)$$



3. Let the angle DCE (Fig. 2) be denoted by b'', and the angles ECF and ECG be equal to each other and denoted by b', then the angle DCF represents the sum of b'' and b', $(b'' + b')$, and the angle DCG the difference between b'' and b', $(b'' - b')$.

Describe from C as centre with the radius CD the arc DF, drop from F the perpendicular FL, and from G the perpendicular GM upon CD, the following sines are represented with respect to the radius CD,

$$\sin(b'' + b') = FL$$

$$\sin(b'' - b') = GM.$$

Draw through E the line IH perpendicular to CE and produce CD, CG and CF to meet IH, respectively, in H, K and I, then the following tangents are represented with respect to the radius $CE = CD$,

$$\begin{aligned} \tan b'' &= EH \\ \tan b' &= EI \\ \therefore \tan b'' + \tan b' &= EH + EI = HI \end{aligned}$$

$$\begin{aligned} \tan b'' &= EH \\ \tan b' &= EK \\ \therefore \tan b'' - \tan b' &= EH - EK = HK. \end{aligned}$$

Draw from I the line IN, and from K the line KQ both perpendicular to CD, then the triangle INH is similar to KQH, and we have

$$IN : IH :: KQ : KH$$
$$\therefore IN : KQ :: IH : KH.$$

But, because the triangle INC is similar to FLC, and KQC similar to GMC; we have

$$\begin{array}{rcccccc} IN & : & FL & :: & CI & : & CF \\ \text{and } KQ & : & GM & :: & CK & : & CG \\ & & \multicolumn{5}{l}{\text{and because } CI = CK, \text{ and } CF = CG} \\ \text{we have } IN & : & FL & :: & KQ & : & GM \\ \therefore IN & : & KQ & :: & FL & : & GM \\ \text{but from the above } IN & : & KQ & :: & IH & : & KH \\ \therefore IH & : & KH & :: & FL & : & GM \end{array}$$

$$\left.\begin{array}{l} \text{that is } (\tan b'' + \tan b') : (\tan b'' - \tan b') :: \sin(b'' + b') : \sin(b'' - b') \\[1ex] \text{or } \dfrac{\tan b'' + \tan b'}{\tan b'' - \tan b'} = \dfrac{\sin(b'' + b')}{\sin(b'' - b')} \end{array}\right\} \quad \ldots\ldots (2)$$

II.

SOLUTION OF THE RIGHT-ANGLED SPHERICAL TRIANGLE.

Let the spherical triangle BRA', (Fig. 3, i. & 4, i.) be a right-angled one of which the spherical angle BRA' is a right angle and $A'BR$ as well as $BA'R$ is an acute angle, then the arc $BA' = d'$ is the hypotenuse, $BR = a'$ the leg opposite to its angle A' and adjacent to its angle B, but $RA' = b'$ the leg opposite to its angle B and adjacent to its angle A'.

Joining CB, CR & CA', the plane angles of which the angular points are at the centre of the sphere and which subtend, respectively, the sides of the spherical triangle are represented by the angles $RCB = a'$, $RCA' = b'$ and $BCA' = d'$.

— Fig. 3, i. —

To represent the measure of the spherical angle B by a straight-lined angle, viz. Art. 11, we draw $A'N'$ perpendicular to CR, $N'(B)$ perpendicular to CB, and join $(B)A'$, then the acute angle (B) opposite to the leg $N'A'$ of the right-angled plane triangle $A'N'(B)$ which has its right angle at N' is equal to the spherical angle B.

— Fig. 4, i. —

To represent the measure of the spherical angle A' by a straight-lined angle, viz. Art. 12, we draw Bn' perpendicular to CR, $n'(A')$ perpendicular to CA', and join $(A')B$, then the acute angle (A') opposite to the leg $n'B$ of the right-angled plane triangle $Bn'(A')$ which has its right angle at n' is equal to the spherical angle A'.

To reason out the relations between the sides and angles of the adopted right-angled spherical triangle by applying the solution of right-angled plane triangles we place BCA' and RCA', sectors of great circles, as well as, respectively, the right-angled plane triangle $A'N'(B)$ or $Bn'(A')$ upon the plane of the sector BCR as follows.

— Fig. 3, i. and Fig. 3, ii. —

Turn the sector BCA' about CB, as upon a hinge, so as to fall upon the plane of the sector BCR and so that they are adjacent to each other, then the lines $(B)A'$ and $(B)N'$ are in the same straight line, because both are perpendicular to CB. In the same manner turn the sector RCA' about CR and the plane triangle $A'N'(B)$ about $N'(B)$ into the position which Fig. 3, ii. shews, so that they all form with BCR but one and the same plane.

— Fig. 4, i. and Fig. 4, ii. —

Turn first the sector BCA' about CA', as upon a hinge, so as to fall upon the plane of the sector RCA' and so that they are adjacent to each other; then turn BCA' and RCA' together about CR, as well as the plane triangle $Bn'A'$ about $n'B$ into the position which Fig. 4, ii. shews, so that they all form with BCR but one and the same plane.

The lines $(A')B$ and $(A')n'$ shall be in the same straight line, because both are perpendicular to CA'.

Fig. 3, i.

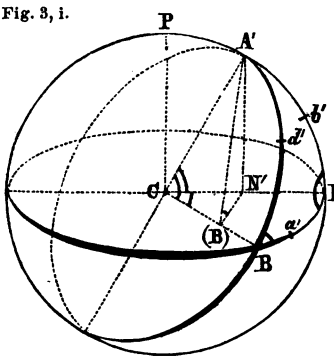


Fig. 4, i.

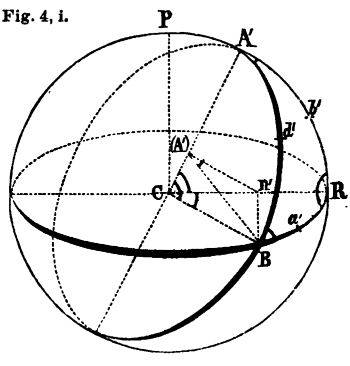


Fig. 3, ii.

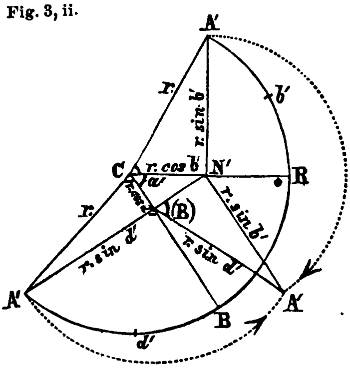
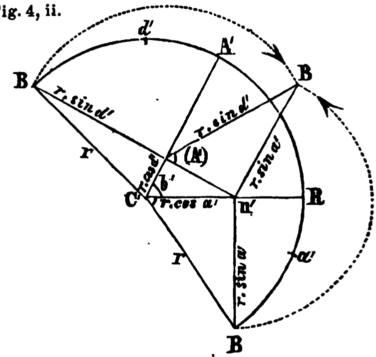


Fig. 4, ii.



To simplify the proof of the following formulæ, we have indicated the values of the legs of the plane triangles of which the radius of the sphere is the hypotenuse, as

— Fig. 3, ii. —

in the triangle $CN'A'$,

$CN' = r \,.\, \cos b'$ & $A'N' = r \,.\, \sin b'$;

in the triangle $C(B)A'$,

$C(B) = r \,.\, \cos d'$ & $A'(B) = r \,.\, \sin d'$;

— Fig. 4, ii. —

in the triangle $Cn'B$,

$Cn' = r \,.\, \cos a'$ & $Bn' = r \,.\, \sin a'$;

in the triangle $C(A')B$,

$C(A') = r \,.\, \cos d'$ & $B(A') = r \,.\, \sin d'$;

(In these expressions the point is chosen as sign of Multiplication, whereas in solving the following equations, Multiplication is indicated by the sign $\times$. In the results there are no points as signs of Multiplication, because the factor r is cancelled as equal factor in numerator and denominator.)

wherefore we have only yet to consider, respectively, the solution of the plane triangles $(B)N'A'$ and $N'(B)C$. | $(A')n'B$ and $n'(A')C$.

$$\frac{A'N}{(B)A'} = \frac{r \,.\, \sin b'}{r \,.\, \sin d'} = \sin (B)$$

$$\therefore \underline{\sin b' = \sin (B) \times \sin d'} \quad \ldots . \ (1)$$

$$\frac{Bn'}{(A')B} = \frac{r \,.\, \sin a'}{r \,.\, \sin d'} = \sin (A')$$

$$\therefore \underline{\sin a' = \sin (A') \times \sin d'} \quad \ldots \ (2)$$

$$\frac{C(B)}{CN'} = \frac{r \,.\, \cos d'}{r \,.\, \cos b'} = \cos a'$$

$$\therefore \underline{\cos d' = \cos a' \times \cos b'} \quad \ldots . \ (3)$$

$$\frac{C(A')}{Cn'} = \frac{r \,.\, \cos d'}{r \,.\, \cos a'} = \cos b'$$

$$\therefore \underline{\cos d' = \cos b' \times \cos a'} \quad \ldots . \ (3)$$

$$\cot (B) = \frac{(B)N'}{N'A'} = \frac{r \,.\, \cos b' \times \sin a'}{r \,.\, \sin b'}$$

$$\therefore \underline{\cot (B) = \cot b' \times \sin a'} \quad \ldots . \ (4)$$

$$\cot (A') = \frac{(A')n'}{n'B} = \frac{r \,.\, \cos a' \times \sin b'}{r \,.\, \sin a'}$$

$$\therefore \underline{\cot (A') = \cot a' \times \sin b'} \quad \ldots . \ (5)$$

$$\cos (B) = \frac{(B)N'}{(B)A'} = \frac{r \,.\, \cos b' \times \sin a'}{r \,.\, \sin d'}$$

but (3) gives $\cos b' = \dfrac{\cos d'}{\cos a'}$

$$\therefore \cos (B) = \frac{\cos d' \times \sin a'}{\sin d' \times \cos a'}$$

$$\therefore \underline{\cos (B) = \cot d' \times \tan a'} \quad \ldots . \ (6)$$

$$\cos (A') = \frac{(A')n'}{(A')B} = \frac{r \,.\, \cos a' \times \sin b'}{r \,.\, \sin d'}$$

but (3) gives $\cos a' = \dfrac{\cos d'}{\cos b'}$

$$\therefore \cos (A') = \frac{\cos d' \times \sin b'}{\sin d' \times \cos b'}$$

$$\therefore \underline{\cos (A') = \cot d' \times \tan b'} \quad \ldots . \ (7)$$

$$\cos (B) = \frac{(B)N'}{(B)A'} = \frac{r \,.\, \cos b' \times \sin a'}{r \,.\, \sin d'} \qquad \frac{Bn'}{(A')n'} = \frac{r \,.\, \sin a'}{r \,.\, \sin d'} = \sin . \,(A')$$

$$\cos (B) \times \frac{\sin a'}{\sin d'} = \sin (A') \times \frac{\cos b' \times \sin a'}{\sin d'}$$

$$\therefore \underline{\cos (B) = \sin (A') \times \cos b'} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (8)$$

$$\frac{A'N'}{(B)A'} = \frac{r \,.\, \sin b'}{r \,.\, \sin d'} = \sin B \qquad \frac{(A')n'}{(A')B} = \frac{r \,.\, \cos a' \times \sin b'}{r \,.\, \sin d'} = \cos (A'')$$

$$\therefore \frac{\sin b'}{\sin d'} \times \cos (A') = \sin B \times \frac{\cos a' \times \sin b'}{\sin d'}$$

$$\therefore \underline{\cos (A') = \sin B \times \cos a'} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (9)$$

$$\frac{(B)N'}{N'A'} = \frac{r \,.\, \cos b' \times \sin a'}{r \,.\, \sin b'} = \cot B \qquad \frac{(A')n'}{n'B} = \frac{r \,.\, \cos a' \times \sin b'}{r \,.\, \sin a'} = \cot (A')$$

$$\frac{\cos b' \times \sin a' \times \cos a' \times \sin b'}{\sin b' \times \sin a'} = \cot (A') \times \cot (B)$$

$$\therefore \cos b' \times \cos a' = \cot (A') \times \cot (B)$$

but (3) gives $\cos b' \times \cos a' = \cos d'$

$$\therefore \underline{\cos d' = \cot (A') \times \cot (B)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (10)$$

Remarks on the Constructions made Art. 43, 54 *&* 61.

1. The construction to find the inclination of the great circle to the Equator given Art. 43 is proved by Fig. 3, although the angle required, respectively, B and (B), is made on different places in Fig. 21 and Fig. 3, ii. In both these Figures, the angle required is opposite to homologous legs of similar right-angled plane triangles.

2. The construction to find the spherical Course at the ship's place Art. 54 is proved by Fig. 4, although the angle required, respectively, A' or A'' and (A'), is made on different places in Fig. 25 and Fig. 4, ii. In both these Figures, the angle required is opposite to homologous legs of similar right-angled plane triangles.

3. To prove the construction of the distance Art. 61, Fig. 21, it is only to be considered that in this figure the line $N'5$ is the same as $N'(B)$ in Fig. 3, ii.

4. To prove the construction of the distance Art. 61, Fig. 25, it is only to be considered that in this Figure the line $n'9$ as also $n''11$ is the same as $N'(A')$ in Fig. 4, ii.

III.

RELATION BETWEEN THE LATITUDES OF TWO GIVEN PLACES AND THEIR LONGITUDES RECKONED FROM THE INTERSECTION OF THE EQUATOR WITH THE GREAT CIRCLE WHICH PASSES THROUGH THE TWO GIVEN PLACES.

Let A'' and A' (Fig. 5) as well as A'' and (A') (Fig. 6) be two given places, and the curve $180 . A''BA'$ the projection of the great circle which passes through them, then o/B indicates one of

Fig. 5.

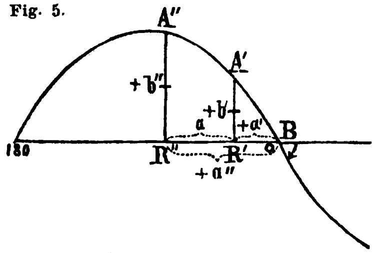
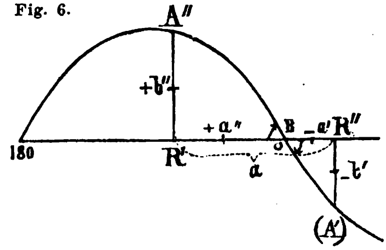


Fig. 6.



the points of intersection of that great circle with the Equator, and the angle B the inclination of that circle to the Equator. The latitudes of the given places are represented in Fig. 5, by $+b''$ and $+b'$, in Fig. 6, by $+b''$ and $-b'$, and the respective longitudes reckoned from o/B in Fig. 5, by $+a''$ and $+a'$, in Fig. 6, by $+a''$ and $-a'$. Because the Diff. Long. between two given places is always the same, even if the two longitudes are reckoned from different origins, a which denotes in each of the Figs. 5 & 6 the Diff. Long. is equal, respectively, to the Diff. Long. given, we have

if the given places are

on the same side of the Equator, Fig. 5,	on opposite sides, Fig. 6,
Diff. Long. . . . . . $= a = +a'' - +a' = a'' - a'$	Diff. Long. . . . . . $= a = +a'' - -a' = a'' + a'$
Sum of the Longs. . . $= +a'' + +a' = a'' + a'$	Sum of the Longs. . . . $= +a'' + -a' = a'' - a'$
Diff. Lat. . . . . $= +b'' - +b' = b'' - b'$	Diff. Lat. . . . . $= +b'' - -b' = b'' + b'$
Sum of the Lats. . . . $= +b'' + +b' = b'' + b'$	Sum of the Lats. . . . . $= +b'' + -b' = b'' - b'$

Wherefore we have only to reason out the formula for one of these cases, and we shall base it upon Fig. 5.

In the right-angled spherical triangle $BR''A''$ is, [Appendix II, formula (4)],

$$\cot B = \cot b'' \times \sin a'' = \frac{\sin a''}{\tan b''},$$

in the right-angled spherical triangle $BR'A'$

$$\cot B = \cot b' \times \sin a' = \frac{\sin a'}{\tan b'}$$

$$\therefore \frac{\sin a''}{\tan b''} = \frac{\sin a'}{\tan b'}$$

or, $\sin a'' : \tan b'' :: \sin a' : \tan b'$

$\therefore \sin a'' : \sin a' :: \tan b'' : \tan b'$ *)

and then $(\sin a'' + \sin a') : (\sin a'' - \sin a') :: (\tan b'' + \tan b') : (\tan b'' - \tan b')$ *)

but [Appendix I, formula (1)] we have got

$$(\sin a'' + \sin a') : (\sin a'' - \sin a') :: \tan \frac{a'' + a'}{2} : \tan \frac{a'' - a'}{2}$$

and [Appendix I, formula (2)] $(\tan b'' + \tan b') : (\tan b'' - \tan b') :: \sin (b'' + b') : \sin (b'' - b')$

$$\therefore \tan \frac{a'' + a'}{2} : \tan \frac{a'' - a'}{2} :: \sin (b'' + b') : \sin (b'' - b')$$

$$\text{which gives } \tan \frac{a'' + a'}{2} = \frac{\tan \frac{a'' - a'}{2} \times \sin (b'' + b')}{\sin (b'' - b')}$$

$$\text{but because } \tan \frac{a'' - a'}{2} = \tan \frac{a}{2}, \text{ and } \frac{1}{\sin (b'' - b')} = \operatorname{cosec} (b'' - b')$$

$$\text{we have } \tan \frac{a'' + a'}{2} = \tan \frac{a}{2} \times \sin (b'' + b') \times \operatorname{cosec} (b'' - b')$$

$$\text{or finally } \log \tan \frac{a'' + a'}{2} = \log \tan \frac{a}{2} + \log \sin (b'' + b') + \log \operatorname{cosec} (b'' - b')$$

Which is the Rule given Art. 46 for finding the sum of the two Longs. required.

To find the two Numbers, here a'' and a', whereof the Sum and the Difference are known, is in other cases of Navigation applied and therefore to be supposed as generally known to seamen.

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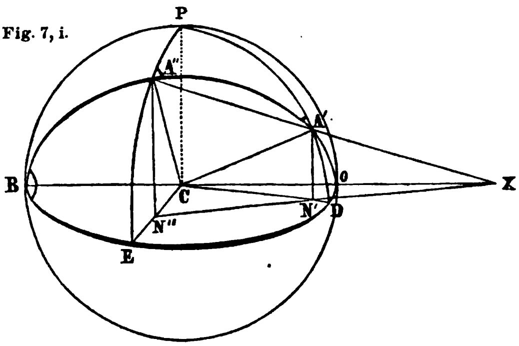
*) The product of the two extremes remains equal to the product of the two means.

IV.

PROOF OF THE CONSTRUCTION
OF THE INTERSECTION OF THE EQUATOR WITH THE GREAT CIRCLE WHICH PASSES THROUGH TWO GIVEN PLACES AND THAT OF THE LONGITUDES OF THE GIVEN PLACES FROM THAT INTERSECTION.

1. Let A' and A'' (Fig. 7, i. and Fig. 8, i.) be two given places on the same side of the Equator the Lats. of which are, respectively, b' and b'', ($b' < b''$), oDEB the Equator, P the respective pole of the Earth, and o$A'A''B$ the great circle which passes through the two given places, then oB the diameter of the Earth, is the line of intersection of the plane of that great circle with the plane of the Equator, o and B are the points of intersection of that circle with the Equat. and the spherical angle B is its inclination to the Equator.

Fig. 7, i.



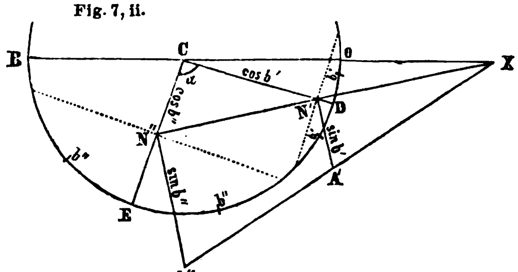
The arcs DA' and EA'' represent the given latitudes; and joining CD, CA', CE and CA'', radii of the Earth, the plane angles DCA' and ECA'' represent also these latitudes, b' and b'', subtending, respectively, those arcs.

The Longs. from the intersection of the great circle with the Equator a' and a'' are represented, respectively, by the arcs oD and oE, as well as by the angles oCD and oCE subtending those arcs.

Finally the Diff. Long. between A' and A'', equal to a, is represented by the arc DE as well as by the angle DCE.

Draw $A'N'$ from A' perpendicular to CD, and $A''N''$ from A'' perpendicular to CE ($A'N'$ and $A''N''$ are perpendicular to the plane of the Equator), then $A'N'$ represents the sine of b', the segment CN' of the radius CD the cosine of b', $A''N''$ the sine of b'' and the segment CN'' of the radius CE the cosine of b'' with respect to the radius of the Earth.

Fig. 7, ii.



Join $A''A'$ and $N''N'$, these two lines are in the plane $N'N''A''A'$ which passes through the two perpendiculars $A'N'$ and $A''N''$, wherefore producing $A''A'$ beyond A', and $N''N'$ beyond N', they meet each other because $b' < b''$.

But the straight line $A''A'$ is in the plane of the great circle which passes through A'' and A', and $N''N'$ is in the plane of the Equator, and both the lines produced remain in those planes; wherefore they must meet each other in the common intersection of these two planes, in the point x of Bo.

Thus, „the point x determines the direction of Bo passing through C, the centre of the Earth."

Now supposing the plane $N'N''A''A'$ to be turned about $N'N''$, as upon a hinge, and to fall upon the plane of the Equator, so as to form with it but one and the same plane, viz. Fig. 7, ii., the point x remains the same as before. — Thus, we have the following

Construction, (Fig. 7, ii.),

to find the point of intersection of a great circle with the Equator and the Longs., a' and a'', reckoned from that point, having given the Lats., b' and b'', ($b' < b''$), and D. Long., a, of two places, A' and A'', — on the same side of the Equator — through which the great circle passes.

With any radius (the chord of 60°, if the construction is to be made by means of a scale of chords) describe a circle or a sufficient part of it, mark its centre C and make the angle $DCE = a$.

Lay off from D, both ways, the Lat. of the place on the right hand, here equal to b', and draw the chord which connects the exterior ends of those two adjacent arcs, then N', the point in which that chord crosses the radius CD, bisects the chord and determines by any half of it the sine b', as well as by the segment CN' of the radius the cos b'.

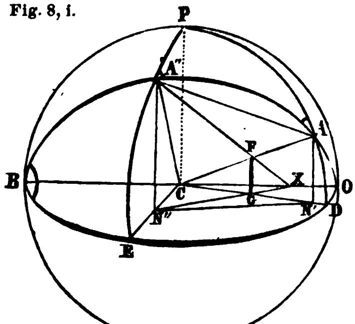
Lay off from E, both ways, the other given Lat., here b'', and join the two exterior extremities of those arcs, then the chord drawn crosses the radius CE in a point N'' which bisects that chord any half of which $=$ sin b'', and cuts off from CE the segment $CN'' =$ cos b''.

Join $N'N''$. — Draw $N'A'$ from N', as well as $N''A''$ from N'' perpendicular to $N'N''$. — Lay off from N' upon $N'A'$ half the chord $=$ sin b' and from N'' upon $N''A''$ half the chord $=$ sin b''. Draw the straight line $A''A'$ and produce it and $N''N'$ to meet each other, then the meeting point X is a point in the direction of the line in which the plane of the great circle and that of the Equator intersect each other.

Draw a line passing through X and C, then the point o in which it cuts the circumference is one of the intersections required, and the arc o$D = a'$, the arc o$E = a''$.

2. The preceding construction excludes not only the case in which the two given places are on the same Parallel of Lat., $b' = b''$, but all cases in which $A''A'$ and $N''N'$ produced do not meet each other on the drawing paper.

Fig. 8, i.

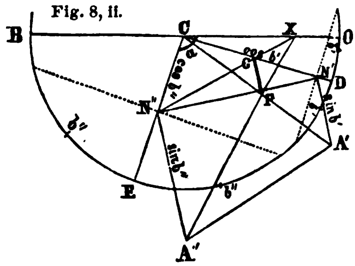


To solve such cases we have found out besides the preceding construction what follows.

Instead of producing $A''A'$ and $N''N'$ (viz. Fig. 7, i.), mark any point, F, on the radius CA' (Fig. 8, i.), and draw from it FG perpendicular to CD, then FG being on the plane of a meridian is also perpendicular to the plane of the Equator in which CD lies. — Draw the lines $A''F$ and $N''G$ produced to meet each other in the point X, then X is on the radius Co or its prolongation, since $A''F$ and $N''F$ must meet each other, because both are on the same plane, $A'N'N''A''$, and $FG < A''N''$, but they must meet each other in a point of Bo, because $A''F$ is on the plane of the great circle and $N''G$ on the plane of the Equator.

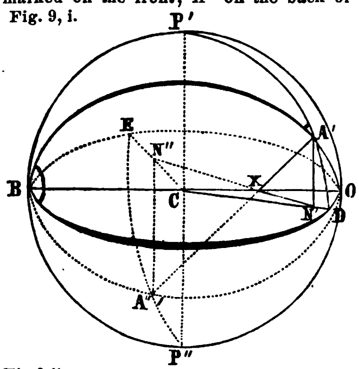
Now, *the solution of the problem to find by construction the point* o is to be finished as follows. After having constructed the four sided figure $A'N'\ N''A''$ (Fig. 8, ii.) as before draw CA', mark on CD the point G (any point on it) and draw through it the line GF parallel to $N'A'$, then GF in Fig. 8, ii. is equal to GF in Fig. 8, i., because CG and CD are in both Figs. the same, the lines $N'A'$ are in both Figs. equal to each other, and in each Fig. the triangle CGD is similar to $CN'A'$. — Draw $N''G$ and $A''F$ produced to meet each other, then the meeting point X is the same as X in Fig. 8, i., because $N''G$ is in both Figs. the same, the lines $N''A''$ are in both Figs. equal to each other, as also the lines GF, and in either Fig. GF is parallel to $N''A''$. — Draw CX produced to meet the circumference, then the meeting point is the point required, o, and $oD = a'$, $oE = a''$.

Fig. 8, ii.



3. Let A' and A'' (Fig. 9, i.) be two given places on opposite sides of the Equator (here A' marked on the front, A'' on the back of the globe represented), P' and P'' the poles of the Earth, and keeping the signification adopted in the preceding $b' =$ arc DA', $b'' =$ arc EA'', $a' =$ arc oD, $a'' =$ arc oE and $a =$ arc $DE =$ angle DCE.

Fig. 9, i.

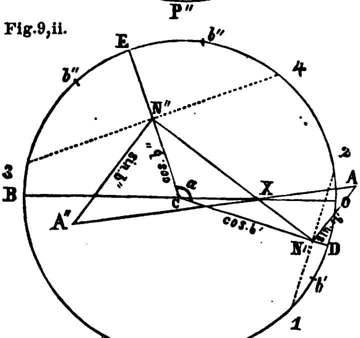


Drawing (Fig. 9, i.), as in case 1. $A'N'$ from A' perpendicular to CD and $A''N''$ from A'' perpendicular to CE and joining $N'N''$ and $A'A''$, these last two lines are in the plane $A'N'N''A''$ and cross each other in a point X which lies on oB, the line of intersection of the plane of the Equator with that of the great circle which passes through A' and A'', because one of these lines is drawn on one of these planes and the other on the other.

Thus, „the point X determines the direction of Bo passing through C, the centre of the Earth".

Turning, as in the first case, the plane $A'N'N''A''$ about $N'N''$, so that it falls upon the plane of the Equator (Fig. 9, ii.) and forms with it one and the same plane $N'N''$ as well as the point X remain in the same position, $N'A'$ and $N''A''$ perpendicular to $N'N''$, $N'A' =$ sin b' half the chord of twice the arc b', and $N''A'' =$ sin $b'' =$ half the chord of twice the arc b''.

Fig. 9, ii.



This leads to the *solution of the problem to find by construction the points of intersection required.* Art. 40 & 41.

With any radius (the chord of 60°, if the construction is to be made by means of a scale of chords) describe a circle (Fig. 9, ii.). — Mark its centre C, and make the angle $DCE = a$. — Lay off from D the arcs $D1 = D2 = b'$; draw the chord 12, and mark the point N' in which it cuts the radius CD. — Lay off from E the arcs $E3 = E4 = b''$, draw the chord 34, and mark the point N'' in which it cuts the radius CE.

Join $N'N''$. — Draw $N'A'$ from N' and $N''A''$ from N'' on the opposite sides of $N'N''$, but both perpendicular to $N'N''$. — Set off $N'A' = N'2$, and $N''A'' = N''3$, and join $A'A''$ which crosses $N'N''$; then the cutting point of these two lines is the point X.

Join CX and produce it, then the cutting points of this line with the circumference are the intersections required, o and B, and the Longs. required are determined, respectively, by the arcs oD and oE.